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CALIBRATION OF AN ELECTROMAGNETIC SEISMOMETER

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled CALIBRATION OF AN ELECTROMAGNETIC SEISMOMETER submitted by Bertrand Alexander Saruk, in partial fulfillment of the requirements for the degree of Master of Science in Geophysics.

ABSTRACT

A method of calibrating an electromagnetic seismometer using a variation in the surrounding magnetic field has been developed. A pair of coils, mounted externally, connected in a series-opposing configuration are used to apply a step in force to the magnet suspended within a Willmore Mk II seismometer. The force is determined from a measurement of the steady state deflection of the magnet with a linear variable differential transformer. The output signal is digitized and recorded on a nine-track, 800 bpi IBM compatible magnetic tape. A z-transform is used to transform the digitized data into the frequency domain. This is then differentiated and normalized so as to give the velocity sensitivity of the seismometer.

This method provides a rapid and accurate method of calibrating the seismometer. Once the coils are mounted no contact is needed with the seismometer so that calibrations can be done regularly and frequently even on seismometers which are not easily accessible.

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CALIBRATION PROCEDURES

1.1 Introduction

Seismic studies involve more than the arrival times and directions of the prominent waves. They require a knowledge of the amplitude and frequency content of the ground motion. It is necessary, then, to determine accurately the characteristics of the system. This requires a calibration technique to determine the magnification or velocity sensitivity and the phase response of the seismometer.

There are several methods employed in obtaining the frequency response of the seismometer from its output. Espinosa et al (1962) used a transient technique which involved taking the ratio of the Fourier transforms of the output and the input transients to yield both the phase and amplitude response of the system in the frequency domain. The input transient used was either a step function or a delta function. Mitchell and Landisman (1968) showed that the direct Fourier analysis gives poor results when microseisms are present and are not negligible in size compared to the output resulting from the calibration transient. They suggested that a least squares fit be used on the seismometer output in the time domain to determine the

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parameters of the seismometer. The frequency response can then be calculated using the transfer function mathematically determined for the system. Certain calibration techniques enable the frequency response to be determined directly from the output by using a sinusoidal input at the desired frequencies (Murphey et al 1952). Though less is involved in processing data in this form, this method is more time consuming than the others because it requires readings at several frequencies for accurate analysis.

The advent of the computer, improved seismometer design and the use of digital recording methods (Burke et al 1970) has given the seismologist more accurate methods of handling data. It has introduced the use of an active seismograph system (use of an amplifier and digital recorder instead of a galvanometer) which permits the seismologist to alter the response of the seismometer system with feedback controlled amplifiers and filters to get the desired response from the system. This has allowed seismologists to attempt studies of recorded pulse shapes as modified along the propagation path, a process which requires an accurate knowledge of the seismometer transfer function.

Deviations of the system parameters from assumed values can lead to serious errors in phase and amplitude corrections. The fact that these parameters may vary due

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to various influences such as temperature and pressure or fatique of the material of the structural element, and that these variations may occur aperiodically or systematically, over a long period of time (one year) or a short period of time (one day) necessitates regular and frequent calibration of the seismometer.

1.2 Calibration Techniques

There are basically three types of seismometers: the moving-coil or moving-mass type, the changing flux type and the magneto-striction type. Shima (1960) pointed out that, in earthquake analysis, the third type is not used and the second type is used to observe only minute and distant earthquakes because of their non-linear response to strong earth motions. Most seismometers are of the first type. It is necessary that a calibration technique be applicable on this type of instrument on a routine and frequent basis.

a) Weight Lifting

Shima (1960) proposed a simple method for determining the voltage sensitivity of a seismometer (figure 1-1). By passing a current (I) through the coil, a force (AI, where A is the motor factor of the seismometer) is exerted on the seismometer mass (M) and displaces it. The seismometer is drawn back up to its

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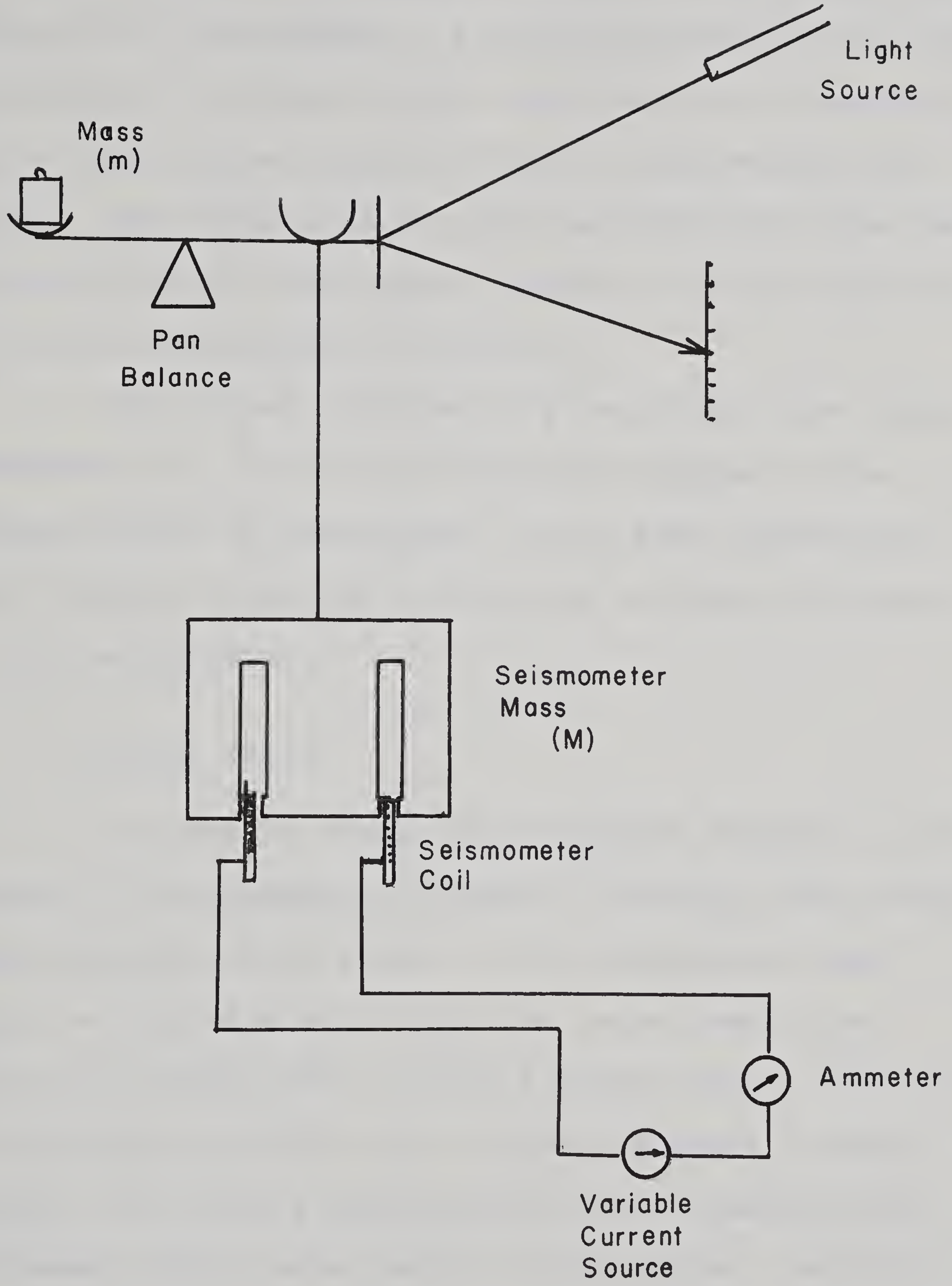

Figure 1-1 Determining the voltage sensitivity of a seismometer by weight lifting technique (Shima, 1960).

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original position by adding a mass (m) to the left hand side of the pan balance. A string attached to the right hand side of the pan balance then pulls the seismometer up to its original position which is determined optically. The value of A can then be determined from the conservation of force which requires that mg = AI where g is the acceleration of gravity.

This method provides only one of the instrument constants, A, and so the frequency response of the system cannot be determined. It is also impractical for frequent field calibrations as it requires access to the seismometer.

b) Shake Table

In order to obtain the frequency response of the system it is necessary to apply a transient and observe the resulting oscillations of the seismometer mass or apply a sine-wave and record the output amplitude. One way of doing this is with a shake table. This is a vibrating platform which can be displaced a given amount at a given frequency and have a constant displacement over a wide range of frequencies. Accurate shake table calibrations require an accurate knowledge of the frequency and magnitude of the shake table vibrations. Improved shake table accuracy has been brought about by the use of piezoelectric crystals (Hoover et al

1965; T. Bardeen 1950) as a driving mechanism by which voltage input is transformed to motion. The transient input of the shake table can be an impulse, step or sinusoidal wave (calculations - Hoover et al 1965).

Shake table calibrations cannot be easily used for frequent field calibrations in that they demand contact with the seismometer which may be buried to reduce temperature and pressure variations. The shake table is also too bulky to be used in field calibrations. It is a useful process by which to initially calibrate the seismometer but it needs to be paired up with another method of calibration for field purposes. Difficulty also arises in using the shake table for the calibration of horizontal seismometers in that a slight tilt in the horizontally vibrating shake table will result in the addition of an external force on the system due to gravity.

c) Weight Adding or Removing

Another technique of seismometer calibration is to add a small mass to or remove it from the moving mass of the seismometer. The removal or addition of a smaller mass m is the equivalent to having a force mg acting on the seismometer. If the removal or addition is done instantaneously it is equivalent to displacing the mass a distance $\Delta x = mg/k$ (where k is the spring constant of

the seismometer) from its rest position and releasing it. As this procedure requires contact with the moving mass of the seismometer it is not useful for field purposes.

A variation on the removal of the mass was suggested by Dix (1952). He proposed the seismometer be placed on a platform supported by a spring having a considerably longer natural period of oscillation than the seismometer and that the platform have a mass suspended from it (figure 1-2a). A sudden removal of the suspended mass results in the seismometer experiencing a force, mg, which can be considered a step where the period of the supporting spring is considerably larger than the length of the record (figure 1-2b). The necessity of the suspended platform makes this a poor procedure for field calibrations.

d) Maxwell Bridge

The seismometer is an electromechanical device. Most procedures thus far have used the mechanical aspect of the seismometer for calibration purposes. Another type of procedure is to apply a force to the moving mass of the seismometer by putting a current through the seismometer coil. Putting a d.c. current through a seismometer coil to displace the mass and then by means of a switch simultaneously shutting off the current and

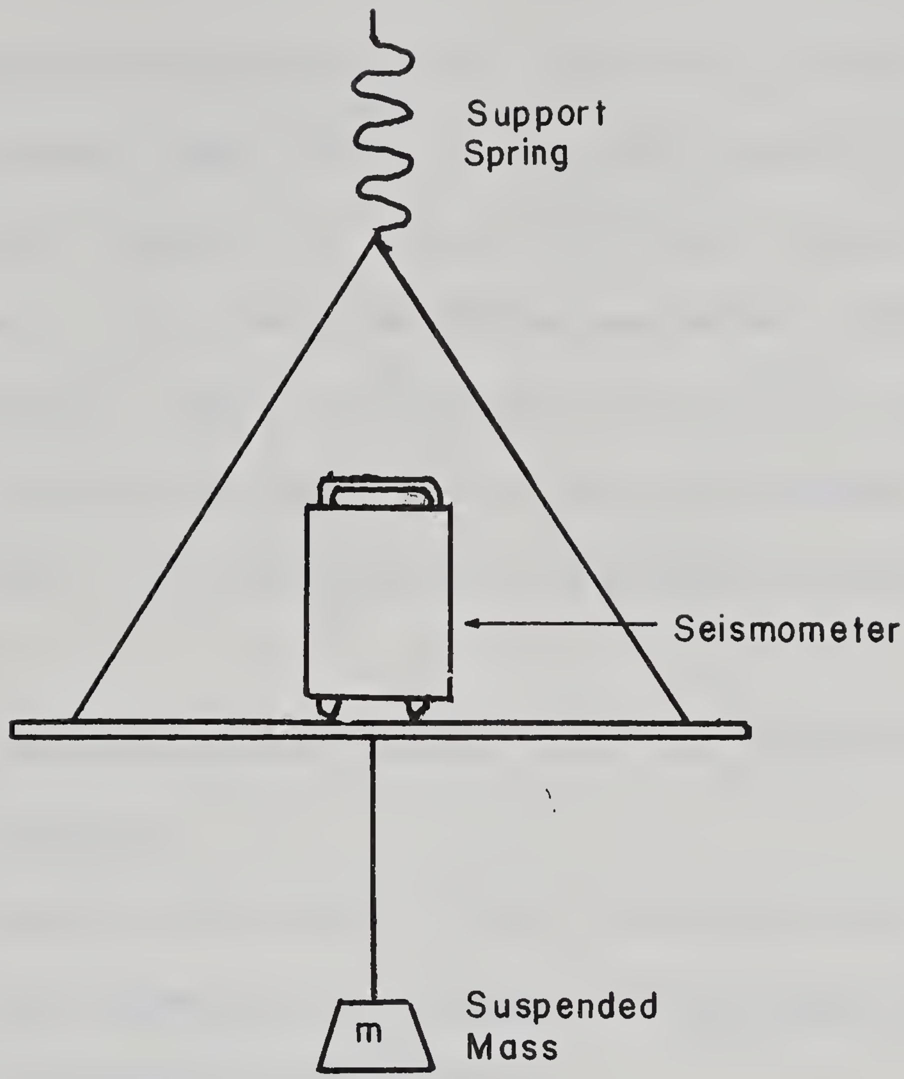

Figure 1-2a Apparatus used in the calibration of the seismometer using step in acceleration (Dix, 1952).

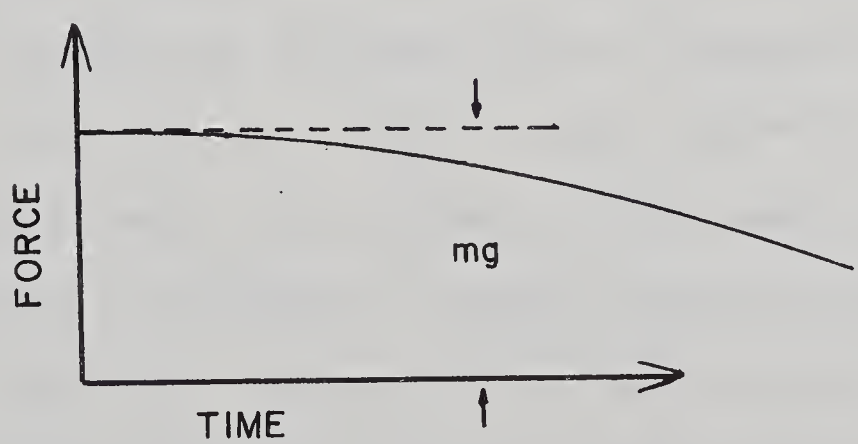

Figure 1-2b Force acting on seismometer due to the removal of mass (m) as a function of time (Dix, 1952).

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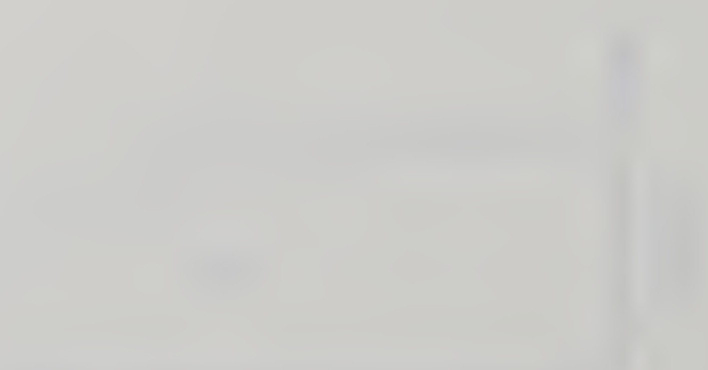

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connecting the seismometer to a recording device (current release test: K.G. Barr 1964; Hoover et al 1965) has been used as a method of field calibration to complement other methods of seismometer calibration.

Willmore (1959) introduced the use of a Maxwell bridge to input the current into the seismometer (figure 1-3a). His calibration procedure consisted of four steps:

1. Balancing the bridge with the seismometer clamped.
2. Reading the size of the deflection of the galvanometer at several frequencies with the seismometer unclamped.
3. Amplitude readings are taken at several frequencies with the seismometer clamped and the bridge on substitution input.
4. Repeat step 3 with the seismometer unclamped.

Any current put through the coil exerts a force, Ai_c, (A - motor factor, i_c - input current) on the seismometer mass. The signal on the measuring device (a galvanometer in Willmore's paper) is then greater than that due to the input current by an amount due to the resulting oscillations of the seismometer mass. Balancing the bridge reduces the input current, as seen by the measuring device, to zero. The observed motion is then entirely that corresponding to a force, Ai_c,

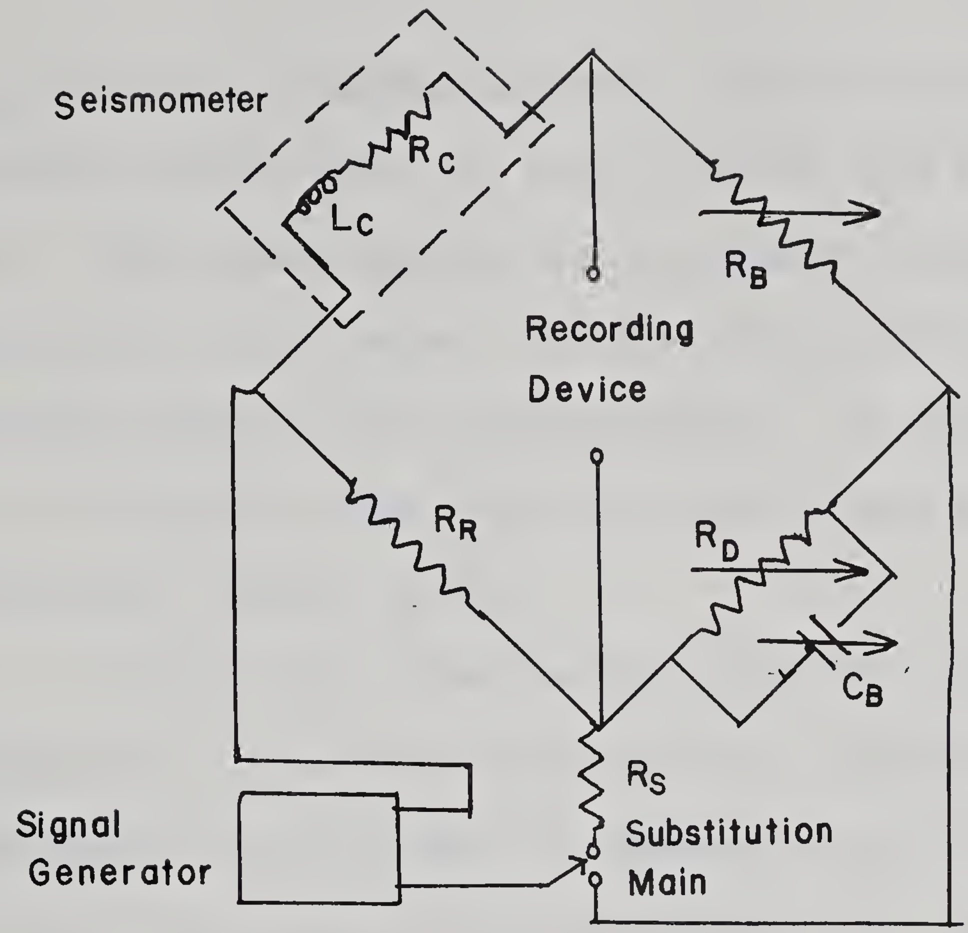

Figure 1-3a Maxwell bridge used in seismometer calibration (Willmore, 1959).

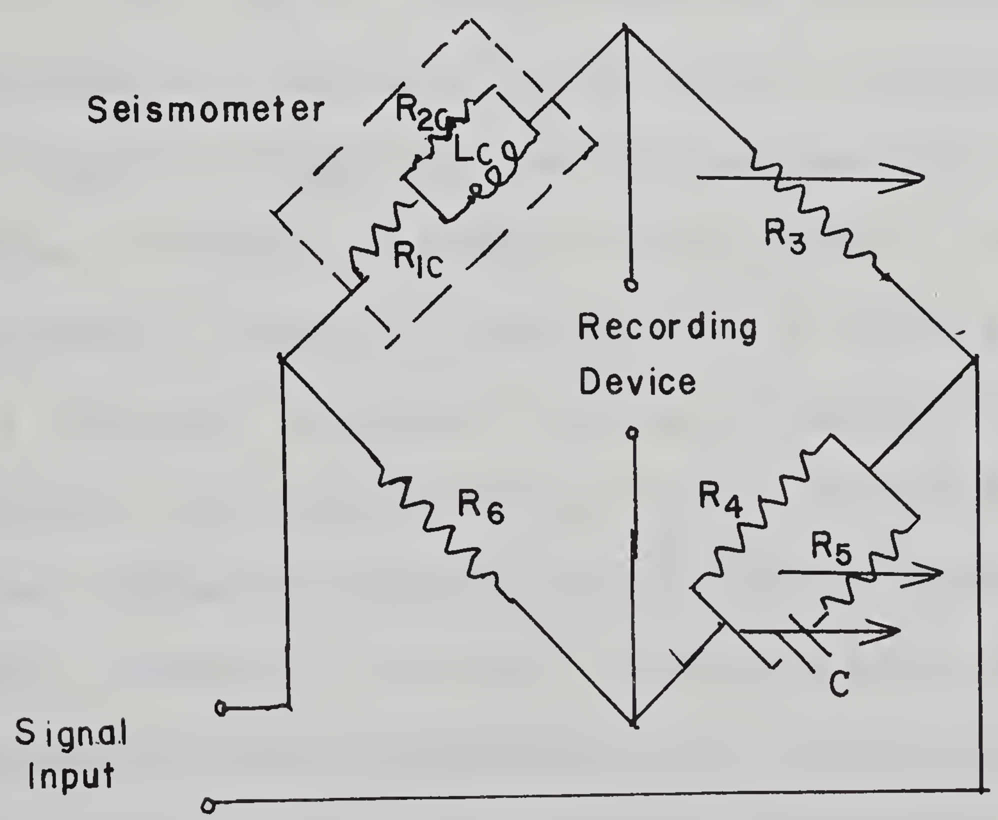

Figure 1-3b Modified Maxwell bridge used in seismometer calibration (Shima et al, 1966).

acting on the seismometer mass. This is equivalent to a ground acceleration of $-Ai_c/M$ (M is the seismometer mass). The substitution input enables an absolute calibration to be made. Though Willmore's original measuring device was a galvanometer, an oscilloscope gives a simpler, more accurate and direct method of calibration (White 1970).

A transient input in the form of a step function - as opposed to the sine-wave used by Willmore - was used by Espinosa et al (1962) to determine the frequency response of a long period seismometer by comparing the Fourier transforms of the output and input signals. At the low frequencies considered, the inductive reactance of the coil can be ignored and the seismometer merely regarded as a resistor which has no frequency dependence. The Maxwell bridge can then be replaced by a Wheatstone bridge. However at higher frequencies, 1 to 20 cps, the seismometer cannot be represented by just a resistor or as a resistor in series with an inductance as done by Willmore. No single balance point can be found unless losses caused by hysteresis and eddy currents are considered. Shima et al (1966) took this into account by representing the seismometer by a resistance in parallel with an inductance and both in series with a second resistance. Their modified Maxwell bridge (figure 1-3b) can then be balanced at one point for frequencies up to

20 cps and the calibration carried out using a step input.

The present calibration procedure at the Department of Physics, University of Alberta takes into account hysteresis and eddy currents by using sine-waves of various frequencies as input and balancing the bridge at every frequency. It has been found that this technique is not suitable for field calibrations in that it requires contact with the seismometer for clamping and unchamping in addition to being a long and tedious method.

e) Calibration Coil

The electric current can also be applied to a separate calibrating coil mounted in, on or around the seismometer. Stewart et al (1967) proposed an external coil for the calibration of a Willmore Mk II seismometer which was mounted on top of the seismometer. The positioning rod of the seismometer is replaced by a connecting rod with a small magnet mounted on the end of it; mass is kept constant. A current passing through the calibrating coil exerts a force on the seismometer mass. The type of forcing function acting on the seismometer depends on the signal put through the coil. The motor function of the calibration coil is determined by the weight-lifting method.

Certain other seismometers have an auxiliary coil built into them for calibration purposes. However this is not always the case. The method of calibration in this thesis is the use of two coils mounted externally to the seismometer with the current through one coil flowing in the opposite direction to the current in the other coil. The two coils are positioned so as to minimize the induced electrical signal. A step function in current through the coils which changes the magnetic field in the volume of the seismometer acting on the suspended magnet is equivalent to a step in the inertial field acting on the seismometer mass. The motor function of this arrangement is determined by measuring the resulting displacement of the mass, $\Delta\ell$, using a linear variable differential transformer mounted on the end of the positioning bar on the Willmore Mk II seismometer. The motor function could also be determined by initially calibrating the seismometer using another technique. This particular method is feasible for frequent field calibrations in that it does not require contact with the seismometer and can quickly and easily be carried out. A sine-wave, step function or impulsive function may be put through the coil.

CHAPTER 2

THE SEISMOMETER TRANSFER FUNCTION

2.1 Transfer Function

The output of the seismometer to an input of a delta function in ground displacement or velocity is known as the displacement or velocity transfer function respectively. The expression for the transfer function can be derived either from an electrical equivalent network of the complete seismograph system (Kollar et al, 1966) or a system of differential equations describing the elements of the seismograph system, as presented in this chapter.

The seismometer used was a Willmore Mk II seismometer (Willmore et al, 1963). It consists of a magnetic mass suspended over a coil by a spring system. This is represented in figure 2-1 with damping represented by the dash pot. x is the displacement of the seismometer case, ξ the displacement of the suspended mass.

When the frame is in motion upwards (positive x) the forces on the mass M are

1) an elastic force (k times the net extension of the spring; k is the elastic constant; ℓ is the rest extension of spring due to seismometer mass)

$$F_e = k(\ell + x - \xi);$$

2) the damping force (retarding α times the velocity of the mass with respect to the frame; α, the damping constant is the sum of viscous and electric damping)

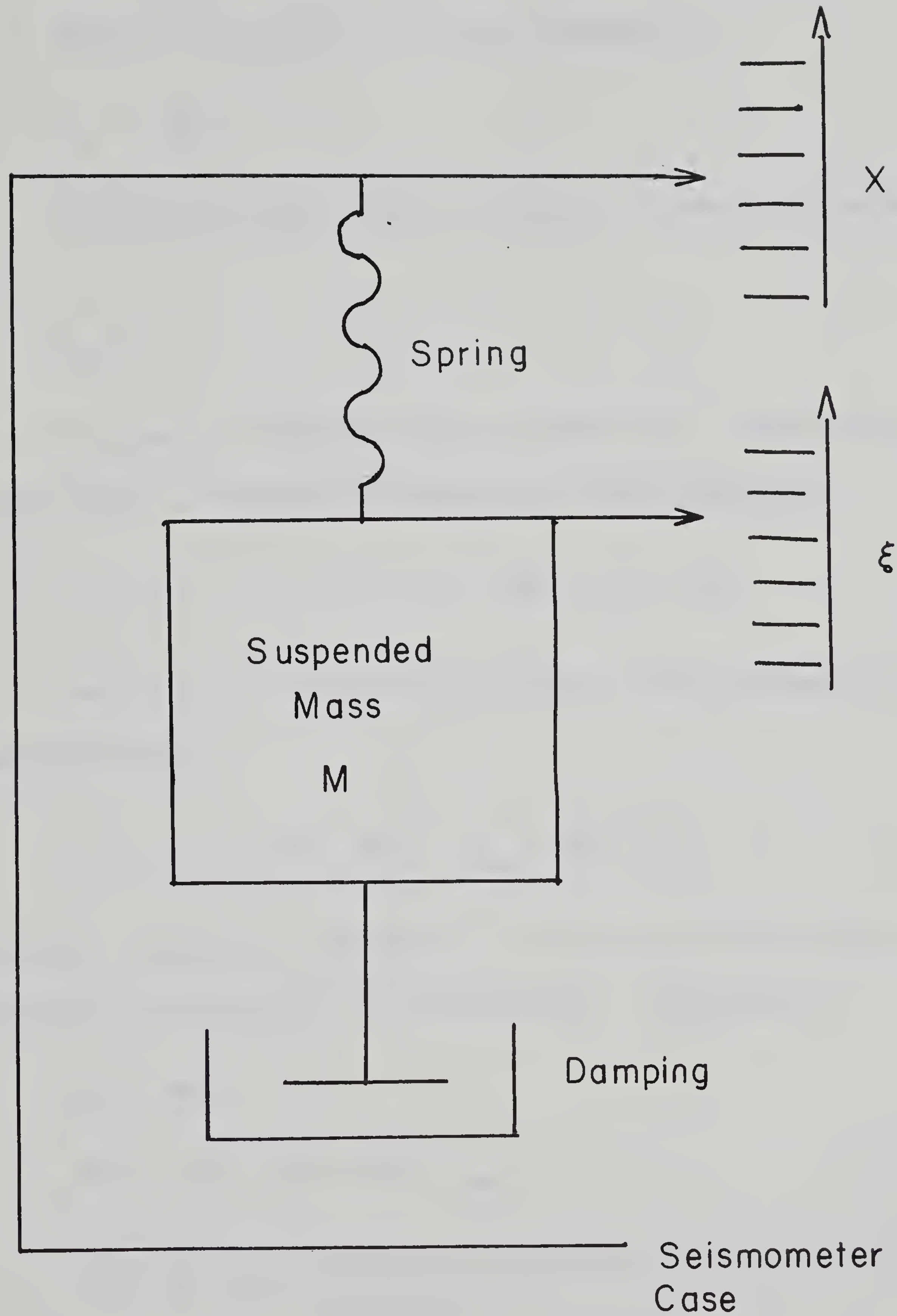

Figure 2-1 The coordinate system of the seismometer. Arrows indicate positive displacement.

$$F_\alpha = -\alpha(\dot{\xi} - \dot{x})$$

3) force of gravity (acting downwards)

$$F_g = -Mg$$

4) magnetic force due to current through external coil

$$F_m \ .$$

The sum of these forces causes an acceleration of the mass $\ddot{\xi}$. Newton's Second Law then requires

$$k(\ell + x - \xi) - \alpha(\dot{\xi} - \dot{x}) - Mg + F_m = M\ddot{\xi} \ .$$

Let $y = x - \xi$ be the relative displacement of the frame and mass

$$k\ell + ky + \alpha\dot{y} - Mg = -F_m + M(\ddot{x} - \ddot{y}) \ .$$

$k\ell$ is the original extension of the spring due only to the force of gravity on the mass M. Therefore

$$k\ell = Mg$$

$$M\ddot{y} + \alpha\dot{y} + ky = M\ddot{x} - F_m \ . \qquad (2.1)$$

$$\omega_n^2 = \frac{k}{M} \quad (\omega_n = \text{natural period of the seismometer in radians})$$

$$\frac{2\zeta}{\omega_n} = \frac{\alpha}{k} \quad (\zeta = \text{damping factor})$$

$$y + \frac{2\zeta}{\omega_n}\dot{y} + \frac{\ddot{y}}{\omega_n^2} = \frac{\ddot{x}}{\omega_n^2} - \frac{F_m}{k} \quad \text{(equation of motion)} \ .$$

Case 1: No magnetic force (i.e. $F_m = 0$)

Taking the Laplace transform of the equation of motion

$$Y(s) + \frac{2\zeta}{\omega_n} \cdot s \cdot Y(s) + \frac{s^2}{\omega_n^2} Y(s) = \frac{s^2}{\omega_n^2} X(s) .$$

The seismometer is a physically realizable instrument so the Laplace transform can be used to solve the differential equation. The Laplace transform of y(t) is Y(s). Normalize this expression using $S = \frac{s}{\omega_n}$

$$Y(S) = \frac{S^2}{1 + 2\zeta S + S^2} X(S) \qquad (2.2)$$

the transfer function is then

$$\frac{Y(s)}{X(s)} = \frac{S^2}{1 + 2\zeta S + S^2} = Y_o(S)$$

for a delta function in ground displacement X(S) = 1 and the transfer function is represented by $Y_o(S)$.

The output voltage of the seismometer is represented by

$$e(t) = -A\dot{y}(t) \qquad (A = \text{motor factor})$$

taking the transform

$$E(s) = -As\,Y(s)$$

$$E(S) = -A\omega_n S\,Y(S) . \qquad (2.3)$$

If x(t) is the input displacement then $\dot{x}(t)$ is the input velocity

$$v(t) = \dot{x}(t)$$

$$\therefore \quad V(s) = s\,X(s)$$

or $$V(S) = \omega_n S\, X(S) \tag{2.4}$$

substituting equations (2.3) and (2.4) into equation (2.2)

$$\frac{E(S)}{-A\omega_n (S)} = \frac{S^2}{1 + 2\zeta S + S^2} \cdot \frac{V(S)}{\omega_n S}$$

$$\frac{E(S)}{V(S)} = \frac{-AS^2}{1 + 2\zeta S + S^2} = E_o(S) \quad . \tag{2.5}$$

$E_o(S)$ is the velocity transfer function - the transform of the output voltage for a delta function input in velocity.

Case 2: No ground motion and a general function for the magnetic field motion

The magnetic force F_m can be represented by $F_m = kD(t)$ where k is the spring constant and D(t) is the displacement of the mass from its rest position due to the force F_m.

The equation of motion is then

$$y + \frac{2\zeta}{\omega_n}\dot{y} + \frac{\ddot{y}}{\omega_n^2} = -D(t)$$

or transformed and normalized

$$Y(S)\,[1 + 2\zeta S + S^2] = -D(S)$$

$$Y(S) = \frac{-D(S)}{1 + 2\zeta S + S^2}$$

expressing Y(S) in terms of voltage

$$E(S) = A\omega_n S\, Y(S)$$

therefore

$$D_o(S) = \frac{E(S)}{D(S)} = \frac{+A\omega_n S}{1 + 2\zeta S + S^2} \qquad (2.6)$$

where $D_o(S)$ is the transfer function due to the magnetic force. The velocity transfer function is

$$E_o(S) = \frac{-AS^2}{1 + 2\zeta S + S^2} \quad .$$

Therefore the velocity transfer function can be derived from the magnetic transfer function.

$$E_o(S) = -\frac{S}{\omega_n} D_o(S) \; .$$

2.2 A Step in Input

Take the case where the magnetic force F_m is due to a step in the magnetic field affecting the seismometer.

$$F_m = kD(t) \qquad t > 0$$

$$F_m = k\Delta\ell \qquad t \geq 0$$

$$= 0 \qquad t < 0$$

$$\therefore \; D(t) = \Delta\ell \qquad t \geq 0$$

$$= 0 \qquad t < 0$$

$$D(s) = \frac{\Delta\ell}{s} = \frac{\Delta\ell}{\omega_n S}$$

Then we get from (2.6)

$$E_{mag}(S) = \frac{+A\Delta\ell}{1 + 2\zeta S + S^2} \quad , \tag{2.7}$$

the velocity transfer function is then

$$E_o(S) = \frac{-S^2}{\Delta\ell} \; E_{mag}(S) \quad .$$

The velocity transfer function can then be obtained from a step in the magnetic field by multiplying the transfer function of the output data by $\frac{S^2}{\Delta\ell}$

$$S = - j \, \frac{\omega_1}{\omega_n}$$

$$S^2 = \frac{-\omega^2}{\omega_n^2}$$

$$\therefore \quad E_o(s) = \frac{+\omega^2}{\omega_n^2} \; \frac{1}{\Delta\ell} \, E_{mag}(s) \quad .$$

The value for $\Delta\ell$ can be obtained by measurement of the output of the linear variable differential transformer.

2.3 A Sine-wave Input

The transfer function can be obtained directly, by using a sine-wave input in the magnetic field. In this case the equation of motion can be rewritten as

$$y(t) + \frac{2\zeta}{\omega_n} \dot{y}(t) + \frac{1}{\omega_n^2} \ddot{y}(t) = -\Delta\ell \sin \omega_1(t) \quad . \tag{2.8}$$

The solution of this differential equation is of the form

$$y(t) = y_p(t) + y_h(t)$$

$$y_p(t) = k_1 \sin \omega_1(t) + k_2 \cos \omega_1(t)$$

substituting back into (2.8) to solve for k_1 and k_2

$$2\zeta \frac{\omega_1}{\omega_n} k_1 \sin \omega_1(t) - (1 - \frac{\omega_1^2}{\omega_n^2}) k_2 \sin \omega_1(t)$$

$$+ 2\zeta \frac{\omega_1}{\omega_n} k_2 \cos \omega_1(t) + (1 - \frac{\omega_1^2}{\omega_n^2}) k_1 \cos \omega_1(t) =$$

$$-\Delta\ell \sin \omega_1(t)$$

Solving for k_1 and k_2

$$y_p(t) = \frac{2\zeta \frac{\omega_1}{\omega_n} \Delta\ell}{(1 + \frac{\omega_1^2}{\omega_n^2})^2 + (2\zeta \frac{\omega_1}{\omega_n})^2} \cos \omega_1(t) +$$

$$\frac{-\Delta\ell \, (1 - \frac{\omega_1^2}{\omega_n^2})}{(1 - \frac{\omega_1^2}{\omega_n^2})^2 + (2\zeta \frac{\omega_1}{\omega_n})^2} \sin \omega_1(t) =$$

$$\frac{\Delta\ell}{\sqrt{(1 - \frac{\omega_1^2}{\omega_n^2})^2 + (2\zeta \frac{\omega_1}{\omega_n})^2}} \sin(\omega_1 t - \theta)$$

where

[illegible]

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$$\theta = \arctan\left(\frac{\zeta \dfrac{\omega_1}{\omega_n}}{1 - \dfrac{\omega_1^2}{\omega_n^2}}\right)$$

$$y_h(t) = k_3 e^{-\omega_n \zeta t} \sin \omega_n \sqrt{1-\zeta^2}\, t + k_2 e^{-\omega_n \zeta t} \cos \omega_n \sqrt{1-\zeta^2}\, t$$

but as we are interested in only the steady state solution and in that case $y_h(t) = 0$

$$y(t) = y_p(t) = \frac{-\Delta \ell}{\sqrt{\left(1 - \dfrac{\omega_1^2}{\omega_n^2}\right)^2 + \left(2\zeta \dfrac{\omega_1}{\omega_n}\right)^2}} \sin(\omega_1 t - \theta) \; .$$

The output from the seismometer is then

$$e_{\sin}(\omega_1) = A\dot{y}(t) = \frac{-\Delta \ell \omega_1 A}{\sqrt{\left(1 - \dfrac{\omega_1^2}{\omega_n^2}\right)^2 + \left(2\zeta \dfrac{\omega_1}{\omega_n}\right)^2}} \cos(\omega_1 t - \theta) \; .$$

We are interested in the velocity transfer function (2.5) which is

$$E_o(s) = \frac{AS^2}{1 + 2\zeta S + S^2}$$

using the substitution

$$S = - j \frac{\omega_1}{\omega_n}$$

$$E_o(\omega_1) = \frac{A \frac{\omega_1^2}{\omega_n^2}}{1 - 2j \frac{\omega_1}{\omega_n} \zeta - \frac{\omega_1^2}{\omega_n^2}}$$

solving for the amplitude and phase of above

$$\text{Amp} = \sqrt{\text{Real}^2 + \text{Imag}^2} = \frac{A \frac{\omega_1^2}{\omega_n^2}}{\sqrt{\left(1 - \frac{\omega_1^2}{\omega_n^2}\right)^2 + \left(2\zeta \frac{\omega_1}{\omega_n}\right)^2}}$$

$$\text{Phase} = \text{A tan}\left(\frac{\text{imag}}{\text{real}}\right) = \text{A tan} \frac{2\zeta \frac{\omega_1}{\omega_n}}{1 - \frac{\omega_1^2}{\omega_n^2}} = \theta$$

The peak to peak output of the sine-wave input is

$$e_{\sin}(\omega_1) = \frac{-2\Delta\ell\omega_1 A}{\sqrt{\left(1 - \frac{\omega_1^2}{\omega_n^2}\right)^2 + \left(2\zeta \frac{\omega_1}{\omega_n}\right)}}$$

and the phase delay of the ouput from the input is

$$\cos(\omega_1 t - \theta) = -\sin(\omega_1 t - \theta - 90)$$

$$\therefore \quad \text{Phase} = 90 + \theta \quad .$$

The velocity transfer function can be obtained by multiplying the output amplitude of each value of the sine-wave by $\omega_1/2\Delta\ell\omega_n^2$ and the phase by subtracting

90° from the measured phase difference.

The output of the linear variable differential transformer in the sine-wave case is

$$V(t) = Ly(t)$$

$$V(t) = \frac{-\Delta \ell L}{\sqrt{\left(1 - \frac{\omega_1^2}{\omega_n^2}\right)^2 + \left(2\zeta \frac{\omega_1}{\omega_n}\right)^2}} \sin(\omega_1 t - \theta) \ .$$

The peak to peak value is

$$V(t) = \frac{+2\Delta \ell L}{\sqrt{\left(1 - \frac{\omega_1^2}{\omega_n^2}\right)^2 + \left(2\zeta \frac{\omega_1}{\omega_n}\right)^2}} \ .$$

which approaches the value $2\Delta\ell L$ as $\omega_1 \to 0$ and since L is known $\Delta\ell$ can be obtained and the velocity transfer function calculated.

CHAPTER 3

THE CALIBRATING PROCEDURE

3.1 Introduction

The calibrating procedure is to apply a known magnetic force to the suspended magnet (mass) of the seismometer. The suspended magnet (figure 3-1) consists of an open-ended cylinder of Magloy which is a permanent magnet, a magnet keep which makes up the inner pole piece, and an outer pole piece. This geometry results in a radial magnetic field across the air gap in which the seismometer coil moves. The relative motion of the suspended magnet to the coil which is fixed to the seismometer case induces an e.m.f. in the coil. This induced e.m.f. is the seismometer output signal.

3.2 The Calibrating Coils

An external magnetic field is applied to the suspended magnet by putting a current through a pair of coils mounted outside the seismometer. For experimental purposes the coils are supported on a frame about the seismometer (figure 3-2) though in normal operation the coils will be assembled into grooves machined in the seismometer case. This will make the coils an integral part of the case and eliminate the possibility of error

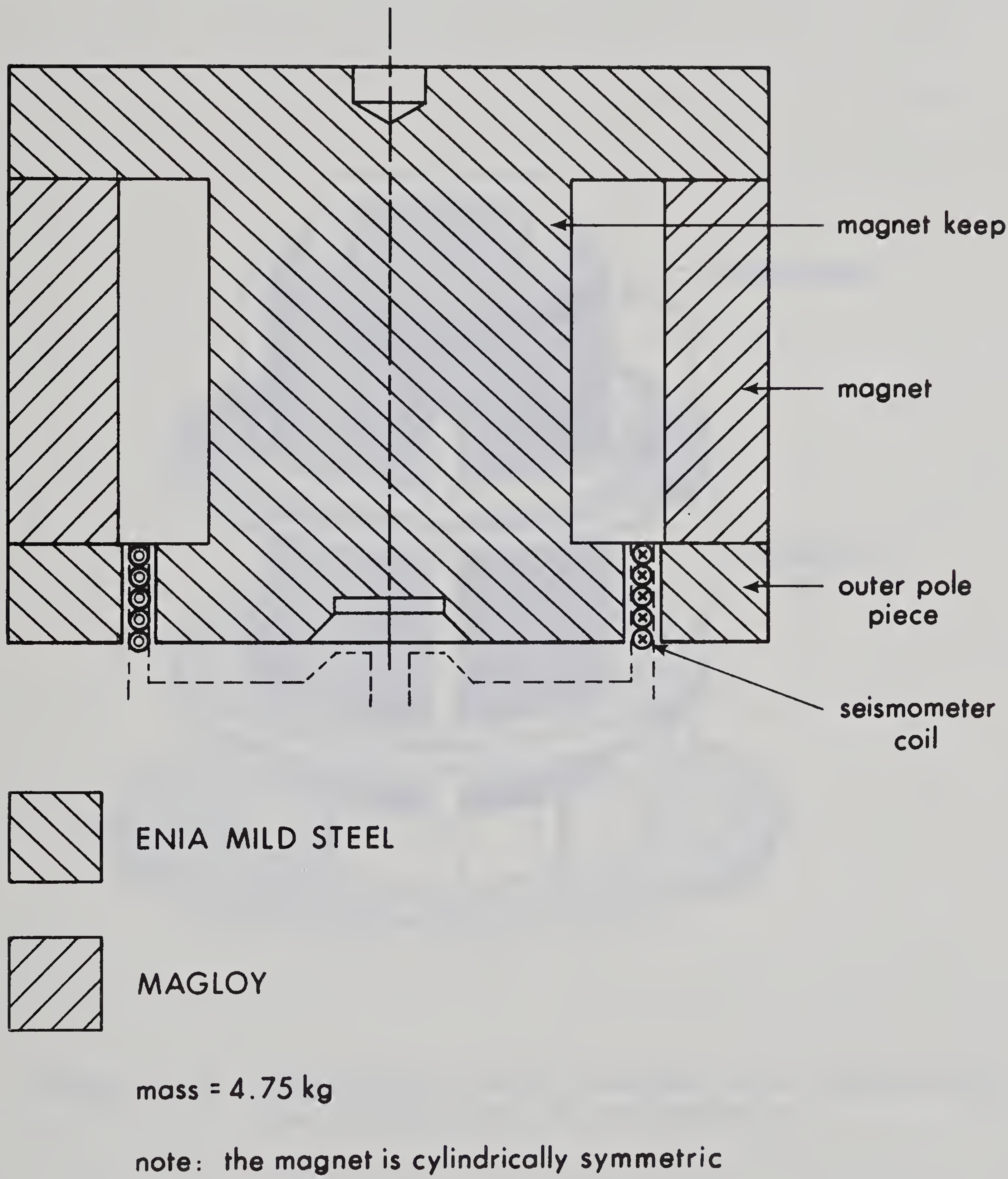
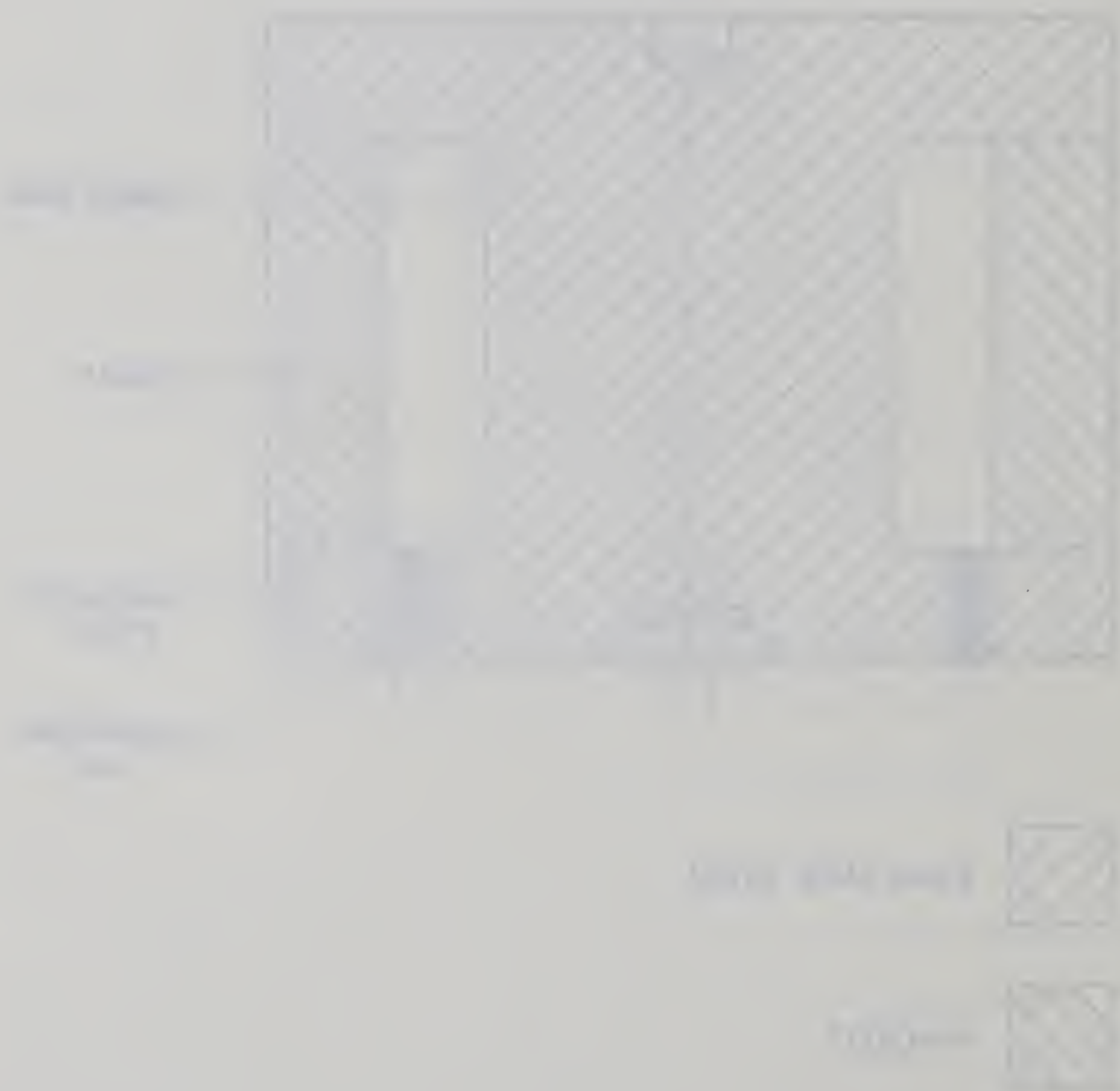


Figure 3-1 The suspended mass (magnet) in a Willmore Mk II seismometer.



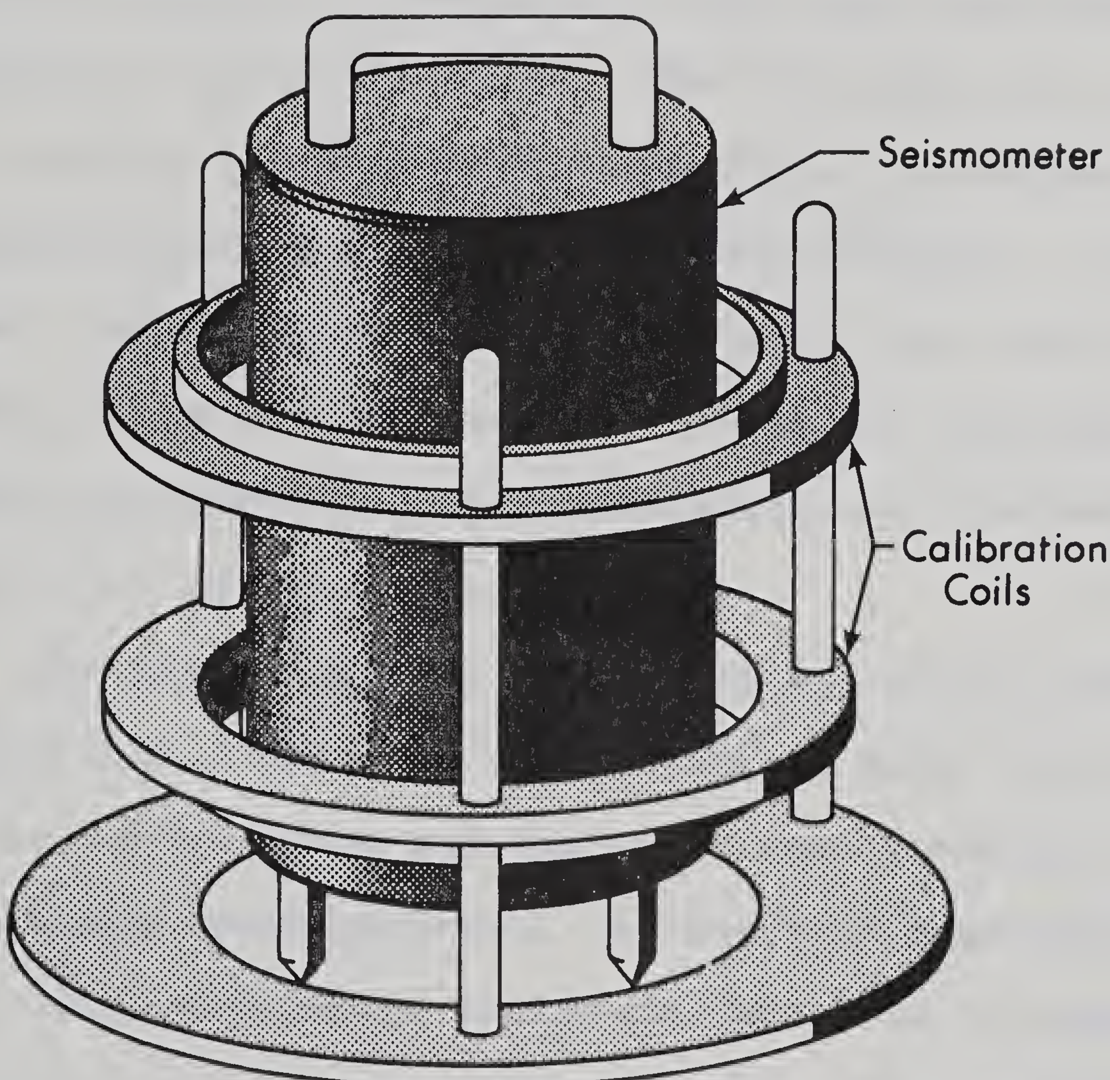


Figure 3-2 Calibration coils mounted on a frame about the Willmore Mk II seismometer.

due to the dependence of input force on the vertical position of the coils with respect to the seismometer mass.

Each coil consists of one hundred turns of number twenty-six A.W.G. copper wire and has a resistance of seven ohms. The coils are wound on bobbins which have an inside diameter of seventeen and a half centimeters and depth of six millimeters. The two coils are connected in series and the calibration signal is applied from a Hewlett-Packard Model 203A function generator. A one kilohm resistor is connected in series with the generator so that it appears as a current source to the load. This reduces the effects of the calibration coil parameters (inductance) on the input.

The two coils were originally set up in a Helmholtz configuration. The coil separation was half the inside diameter of the coils and the coils were connected in a series aiding configuration. A change in magnitude of a near uniform magnetic field was then used to apply the change in force on the suspended mass. The resulting deflections were small resulting in output voltages of less than one volt per ampere of current applied to the coils and large induced e.m.f.'s were observed in both the seismometer coil and the linear variable differential transformer (LVDT).

The direction of current flow in one coil was then reversed so that the coils were now connected in a series

opposing configuration. The deflection of the seismometer in this configuration was greater by several orders of magnitude. The mechanism involved in producing this effect can be seen by examining the magnetic field between the two coils and considering that the translational force acting on a magnet in a magnetic field depends directly on the gradient of the magnetic field in the direction of the force and the component of the magnetic dipole (which represents the suspended magnet) that lies parallel to this gradient. The field in the series aiding configuration is a fairly uniform field and so very little net force acts on the magnet. The series opposing configuration gives a large gradient in the axial direction (illustrated qualitatively in figure 3-3). A larger force then acts on the magnet. Changing the current direction in both coils changes the sign of the gradient and the force acts in the opposite direction.

The non-uniformity of the direction of the magnetic field (it is not all in the axial direction) enables a force to act in the axial direction even if the magnetic moment of the suspended magnetic mass lies in a radial plane. By rotating the reference axis (figure 3-4) so that the magnetic moment no longer lies perpendicular to the field gradient in the direction of force it can be shown a force which has a component in the axial direction then acts on the magnet. The cylindrical symmetry of the

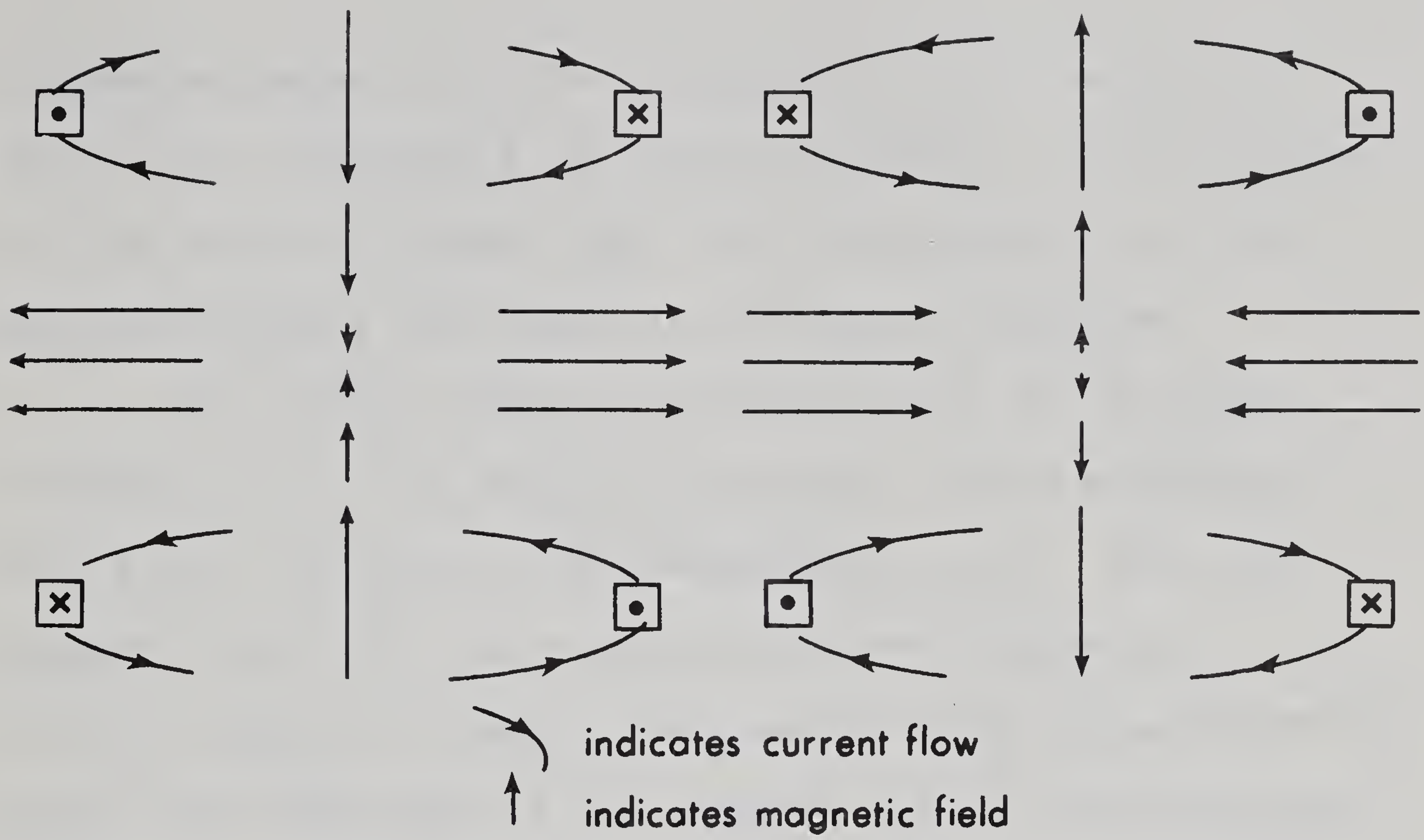


Figure 3-3 Qualitative representation of the magnetic field due to two coils connected in a series opposing configuration.

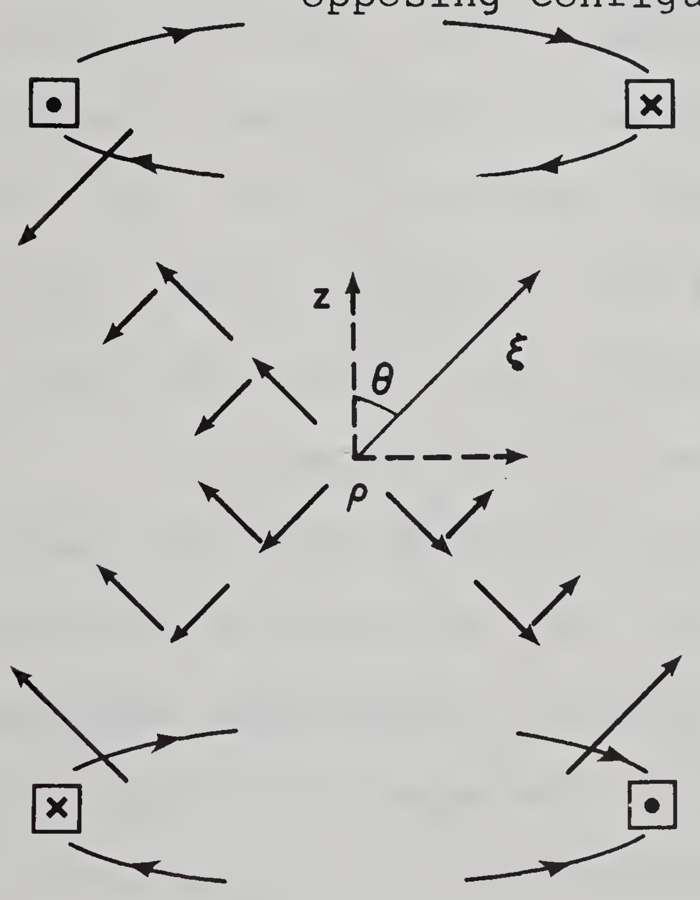


Figure 3-4 Qualitative representation of the magnetic field due to the two coils with reference axis rotated.

suspended magnet and the calibration coils results in the radial components of force cancelling. The location of the magnetic moment away from the midpoint of the coil provides a net-force in the axial direction.

The series opposing configuration can be positioned so that the resulting magnetic field minimizes the e.m.f. induced in the seismometer coil. The coil assembly with the two coils at minimum separation (2.6 cm centre to centre) was moved along the seismometer until the magnitude of the induced e.m.f. was minimized. This was measured by clamping the seismometer and measuring the size of the induced transient with an oscilloscope. The position of minimum spike height was found to be five and a half centimeters from the seismometer base. The calibration coils are now centred about the fixed coil within the seismometer.

The separation of the two calibration coils was then increased about this point and the displacement of the magnet from its rest position was measured. The maximum displacement (maximum force acting on magnet) was found at a separation of 7.6 centimeters. The bottom of the magnet was now entirely within the volume of the coils. The deflection then remained constant for increasing coil separation. The separation used for calibration was 9.6 centimeters. The induced e.m.f. was then measured at 2.5 millimeter intervals around the

earlier determined minimum with this separation of the coils (figure 3-5a). The coil location is the distance from the bottom of the seismometer case to the midpoint of the calibration coils. A new minimum was then located 6 centimeters above the seismometer base. This change in position was due to a magnetic permeable mass (the magnet) being located between the top coil and the seismometer coil. The unclamped response of the seismometer was also measured and figure 3-5b shows the variation in magnet deflection versus coil position.

3.3 Linear Variable Differential Transformer

An absolute calibration requires a knowledge of the force which acts on the suspended mass (magnet). This can be found by measuring the axial displacement, $\Delta \ell$, of the suspended mass. The applied force is then $k\Delta \ell$ where k is the spring constant of the seismometer. The value for $\Delta \ell$ is obtained using a linear variable differential transformer (LVDT). The LVDT used was a 500 DC-D transformer manufactured by Schaevitz engineering. It has a sensitivity of 7.70 ± 0.003 volts per centimeter (figure 3-6) and a range of linearity of ± 1.2 centimeters.

The core of the LVDT and its support rod are mounted in place of the positioning rod of the seismometer and the transducer is mounted on top of the seismometer over the support rod (figure 3-7). The support rod is carefully and rigidly mounted so that

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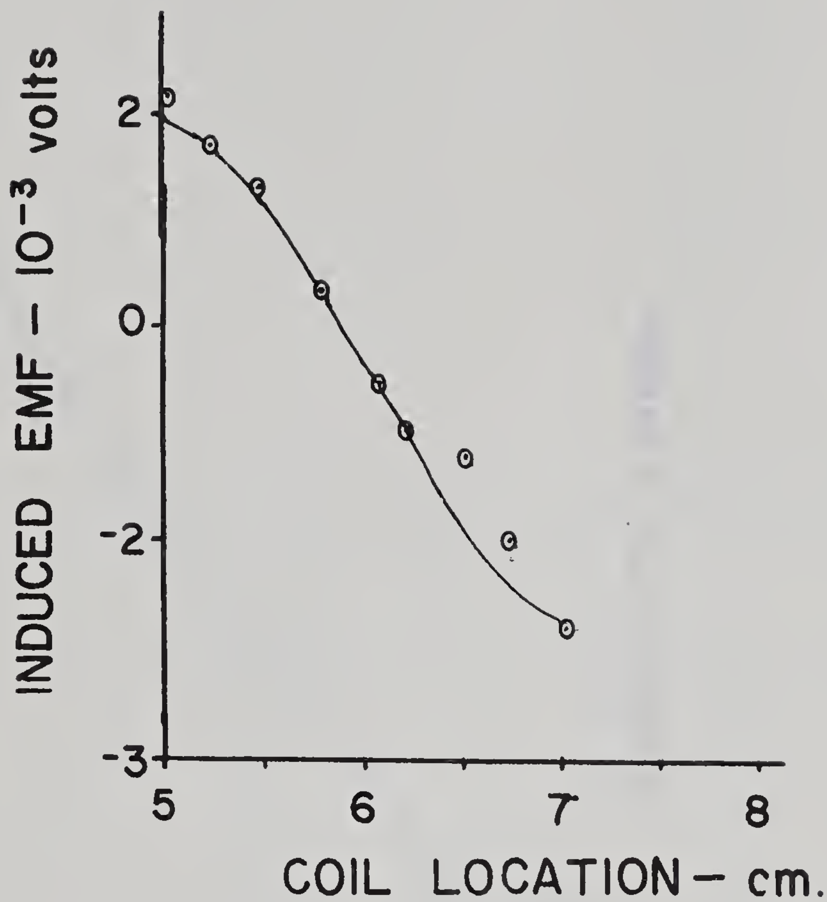

Figure 3-5a E.m.f. induced in seismometer coil for a one ampere current flowing through the calibrating coils measured at various coil locations (see text).

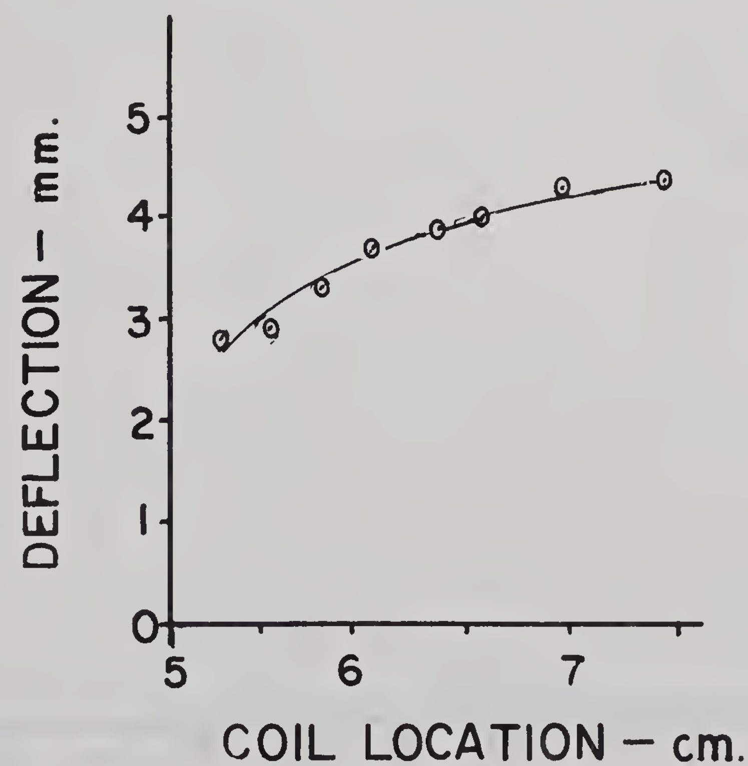

Figure 3-5b Deflection of the seismometer mass in millimeters for a one ampere current put through the calibration coils measured at various coil locations (see text).

Figure 3-6 Calibration curve for linear variable differential transformer used in the seismometer calibration.

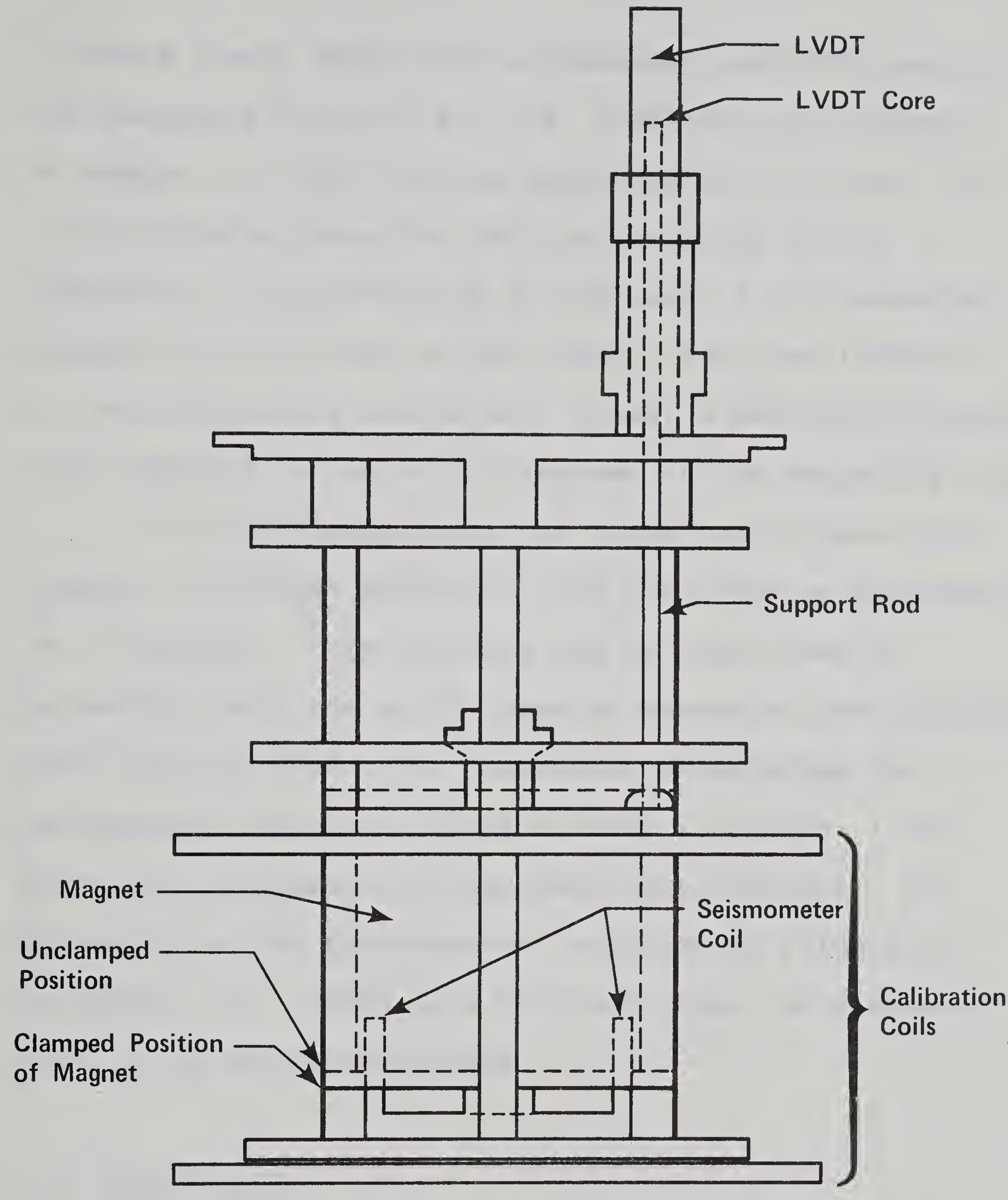

Figure 3-7 The LVDT mounted on a Willmore Mk II seismometer. The position of calibration coils with respect to the seismometer mass, clamped and unclamped, is also illustrated.

it moves freely within the seismometer and LVDT aperture and therefore does not add any damping to the system. No change is noted in the amplitude of the output of the seismometer when the LVDT is operating or not energized. The difference in the mass of the suspended system due to the use of the support and core instead of the positioning rod is 24.5 grams, a negligible amount when compared to the 4.75 kilograms of the suspended mass.

The relationship for the force on the mass with respect to current applied to the coils can be expressed as a constant. This constant can be determined by measuring the force on the mass by measuring the displacement using an LVDT or by originally calibrating the seismometer using one of the methods in Chapter 1 and using the seismometer to determine the constant. The linearity of the force-current relation is illustrated in figure 3-8. Force is a constant times the displacement of the seismometer mass.

3.4 Amplifiers

To accurately record the data the analog output from both the seismometer and LVDT are amplified to have a dynamic range of ±10 volts. Both signals were amplified using 184 L operational amplifiers. The positive input was used so that the value of the load resistor, R_L, in each case would not affect the

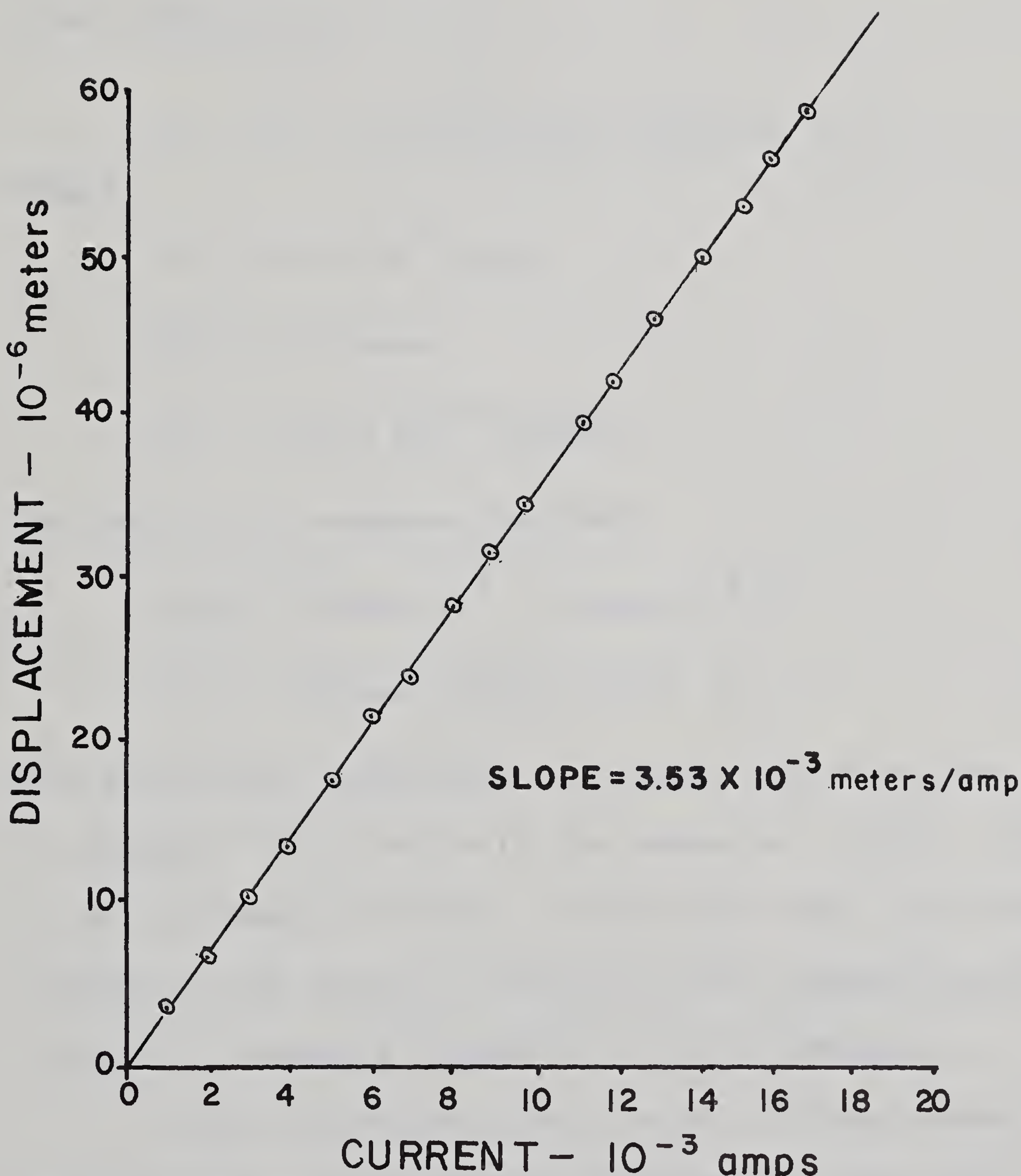

Figure 3-8 Displacement of seismometer mass for various currents put through the calibration coils. The position of the calibration coils is kept fixed at 6 cm above the seismometer base.

frequency response of the amplifier. In the case of the seismometer output it is necessary to know the frequency response of the amplifier used (figure 3-9a). This can be determined by measuring the amplitude and phase response at several frequencies or calibrating these values for the amplifier. The closed loop amplifier response is

$$z = 1.0 + (R1/R2^{*}\times(1.0 + j\omega R1C)$$

where

$$R1 = 4.3\times 10^{5} \text{ ohms}$$

$$R2 = 10^{4} \text{ ohms}$$

$$C = 3.36\times 10^{-9} \text{ farads .}$$

The amplitude response is then

$$Amp = ((\text{Real } z)^{2} + (\text{Imag } z)^{2})^{\frac{1}{2}}$$

$$Phz = \text{Arctan } (\text{Imag } z/\text{Real } z)$$

The calculated amplitude and phase responses are shown in figure 3-10 along with the measured values. The close agreement of these values show that the calculated values can be used to transform the recorded data to the original frequency response of the seismometer.

Only the steady state value of displacement needs to be known so it is only necessary to know the D.C. gain of the LVDT amplifier (figure 3-9b). The calculated and measured values agree at 110±1 for the D.C. gain

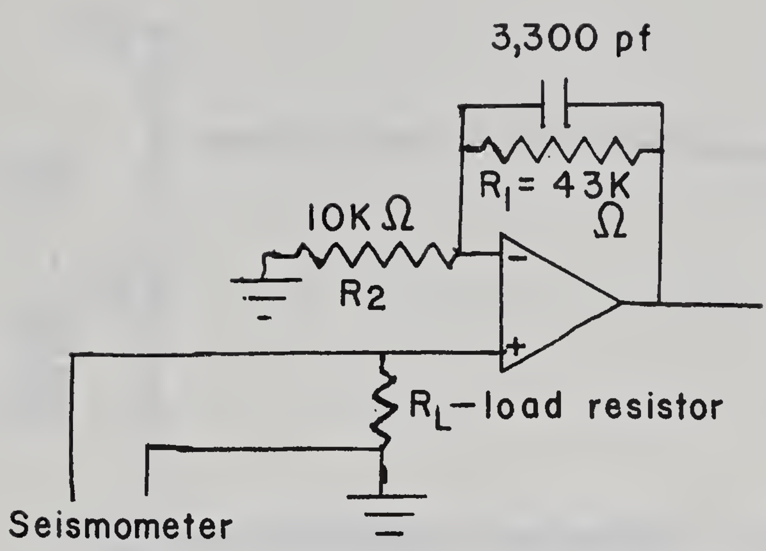

Figure 3-9a The amplifier used to amplify the output signal of the seismometer.

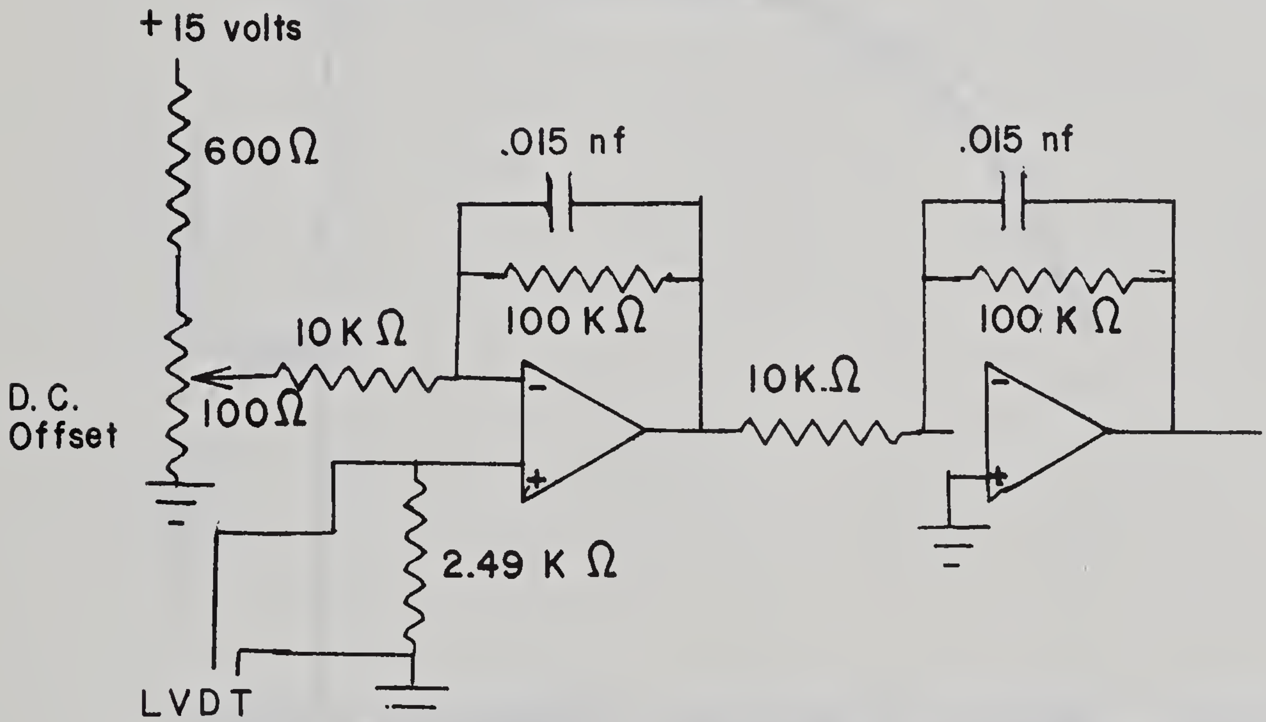

Figure 3-9b The amplifier used to amplify the output signal of the LVDT.

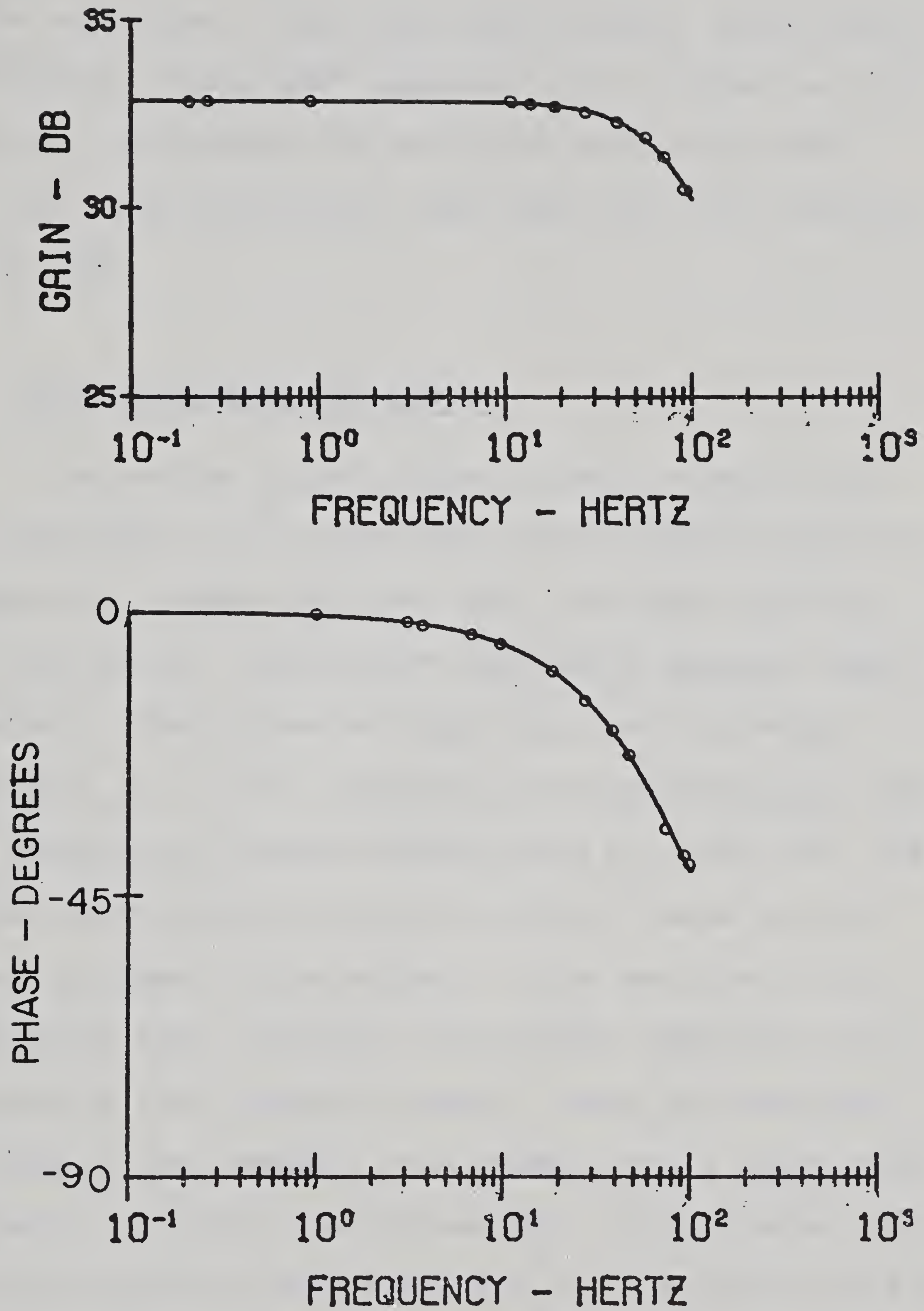

Figure 3-10 Calculated phase and gain of amplifier used on seismometer output. Points marked on curves represent measured values.

of the amplifier. This high gain as well as the high sensitivity of the LVDT requires a D.C. offset on the amplifier to prevent the amplifier saturating when the core oscillates about other than the zero location of the LVDT.

3.5 Data Recording Sub-system

The analog output signals from the amplifiers were digitized so that the data can be manipulated by a computer. These digitized data are then recorded on a nine-track, 800 bpi IBM compatible magnetic tape transport. The system of digitizing and recording (Allsop et al., 1972) consists of a modified model #120 Data Acquisition System manufactured by Datum Inc. and a model 7830 synchronous tape transport made by Peripheral Equipment Corporations. In as much as it is a synchronous tape transport the maximum sampling rate is governed by the transport speed. Since the recording was done on ten channels each channel had a digitizing interval of $10/2500 = 0.004$ seconds. Each channel had a record length of 4096 words and was therefore $0.004 \times 4096 = 16.38$ seconds long. The response to the step input on each record then had adequate time to damp out before the end of the record. The A to D converter has a resolution of 14 bits, including sign, on a full scale input of ± 10 volts.

Channel one was used to record the input step in voltage measured across the calibration coils and channel three used to record the output from the LVDT. The seismometer output is recorded on channels two, four, six, eight and ten and the remaining channels are grounded. The multiplexer sampling interval is 0.0004 seconds to satisfy the synchronous tape input requirements. If the seismometer data are recorded on every n-th channel and then intermixing the data the sampling interval is n times 0.0004. Recording on one of the ten channels then gives a sampling interval of 0.004 seconds and a Nyquist frequency ($f_N = 1/2\Delta t$ where Δt is the sampling interval) of 125 Hz. Recording on every second channel gives a sampling interval of 0.0008 seconds and a Nyquist frequency of 625 Hz.

CHAPTER 4

THE PROCESSING AND EVALUATION OF DATA

4.1 Introduction

The purpose of calibrating the seismometer is to arrive at the frequency response of the seismometer-log amplitude and phase. The frequency response of the seismometer depends on the natural frequency (f_n) of the seismometer, the damping (ζ) acting on the seismometer, and the motor factor (A) of the seismometer magnet-coil configuration. The frequency response is obtained by transforming the data measured in the time domain to the frequency domain, and correcting for the frequency response of the amplifiers and data recording subsystem.

4.2 Calculated Response

The expression for the velocity sensitivity (sensitivity of the seismometer to an impulse in ground velocity) as derived in Chapter 2 (equation 2.5) is

$$E_o(S) = \frac{-AS^2}{1 + 2\zeta S + S^2} \qquad S = -\frac{jf}{f_n}$$

ζ = damping factor

f_n = natural frequency in hertz

A = motor factor of the seismometer in volts per centimeter per second.

Two of the above characteristics are easily altered on the Willmore Mk II seismometer: the damping factor and the natural frequency. Figure 4-1 shows the log amplitude and phase response for $E_o(S)$ at several values of ζ - ranging from $\zeta = 0.3$ (the outer curve) to $\zeta = 1.0$ (the inner curve). The value for A is taken to be 1 volt/(cm/sec) in all cases and f_n is taken to be 0.977 Hz, the value found for the seismometer calibrated. The curves of the log amplitude response have two asymptotes - one which behaves as the s^2 term at low frequencies so rises at 40 db per decade and one which behaves as 1 at high frequencies. The two asymptotes intersect at the natural frequency (f_n).

For $\zeta = 1$ (critical damping) the motion of the suspended mass in the time domain due to an impulse in ground velocity behaves as $D(e^{-\omega_n t} - \omega_n t\, e^{-\omega_n t})$ where D is the magnitude of the inpulse and ω_n is the natural frequency of the seismometer in radians/sec. There is no overshoot in the time domain. The amplitude response in frequency domain is down 6 db from the horizontal asymptote at the natural frequency. A more desirable response is 0.707 critical damping. It is maximally flat at higher frequencies and never exceeds one (flat frequency response). The response is down 3 db at the natural frequency. A damping factor of 0.5 critical has a

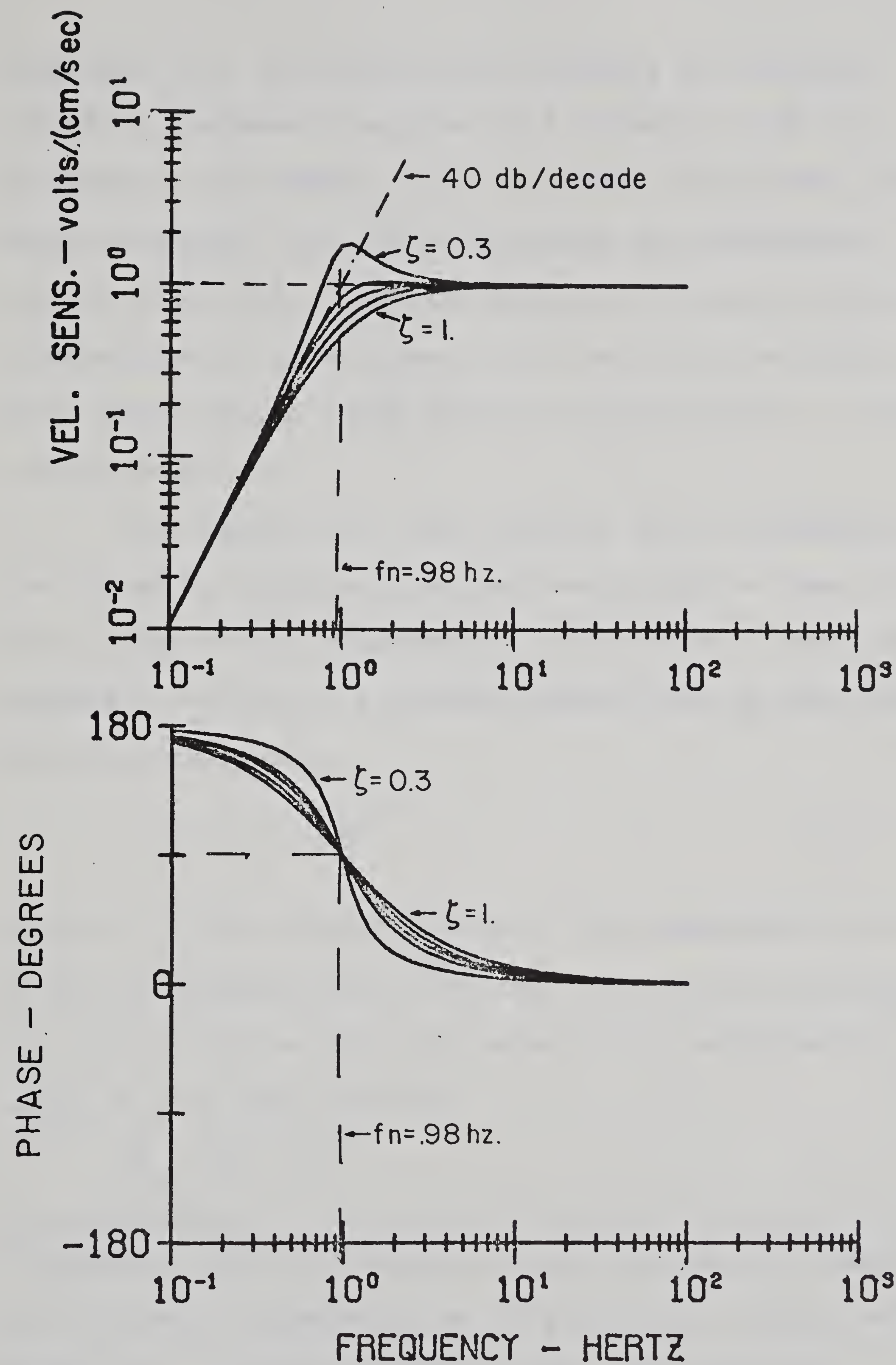

Figure 4-1 Calculated velocity sensitivity and velocity phase of the seismometer. A = 1 volt/(cm/sec), f_n = .98 hz and ζ = 0.3, 0.6, 0.707, 0.8 and 1. Asymptotes are shown for the velocity response.

response of 1 at the natural frequency but exceeds the flat frequency response by 1.25 db at 1.38 Hz. Decreasing the damping made the phase go through 180° more gradually. At low frequencies the seismometer output leads the ground velocity by 180° and at high frequencies it is in phase with the ground velocity. The output always leads ground velocity by 90° at the normal frequency.

The damping for the Willmore Mk II seismometer is varied by changing the load resistance as seen by the output of the seismometer. The value of resistance needed to arrive at a desired damping can be derived from the expression

$$n + 1 = 1.7 \frac{T}{D}^{\dagger} \qquad (4.1)$$

where T is the natural period of the seismometer and D is the damping factor desired. The load resistance R_L, is related to the resistance of the seismometer coil, R_c, by the relation:

$$R_L = R_c \times (n) \ .$$

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† Equation (4.1) is derived in the Instruction Manual for Willmore Seismometry Mk II put out by Hilger and Watts Ltd., London, England, 1964. The relation $n+1 = 1.6 \frac{T}{D}$ is derived in Willmore et al (1963).

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For 0.707 critical damping at a natural period of one second and a coil resistance of 3.3 kilohms a load resistance of 4.29 kilohms is needed. Assumptions made in the derivation of equation (4.1) and the presence of additional resistance in the form of the seismometer cable make this value only an approximation. The calibrations were done using load resistances of 4.11 kilohms, 4.6 kilohms and 10 kilohms.

Figure 4-2 shows the asymptotes of the log amplitude response and the phase response for inputs of a step in ground acceleration, an impulse in ground acceleration (step in ground velocity), and an impulse in ground velocity (step in ground displacement). The expression for the seismometer response for a step in ground acceleration is

$$E_{step}(S) = \frac{C}{1 + 2\zeta S + S^2} \qquad \text{(equation 2.7)}$$

where C is a constant. At low frequencies it behaves as 1 and at high frequencies it behaves as $1/S^2$. This yields two asymptotes - one which decreases at 40 db per decade and one which has the value amplitude = 1. The two asymptotes intersect at the natural frequency. Each differentiation of the input signal causes a rotation of +20 db/per decade (multiplying the amplitude response by the normalized frequency f/f_n).

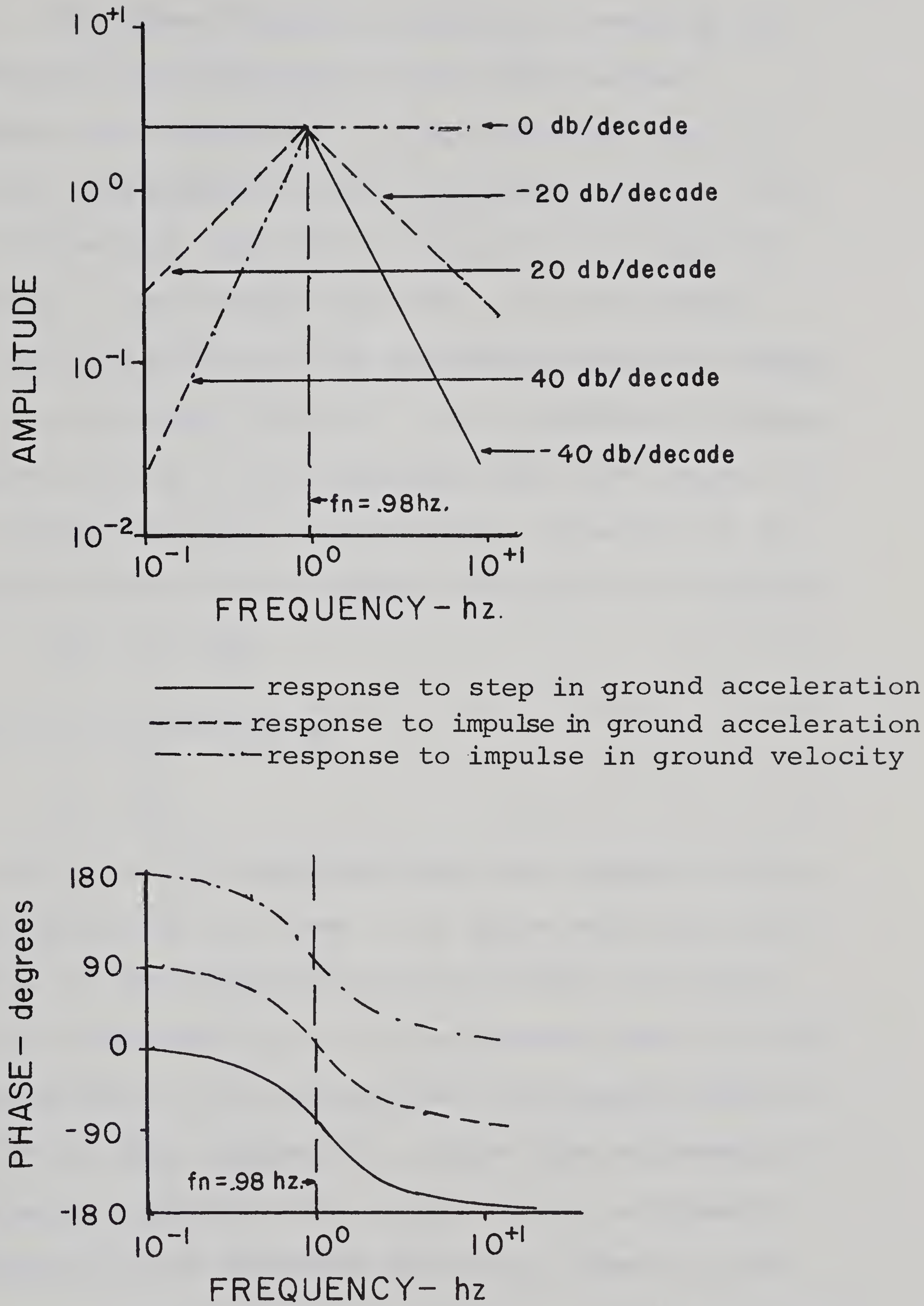
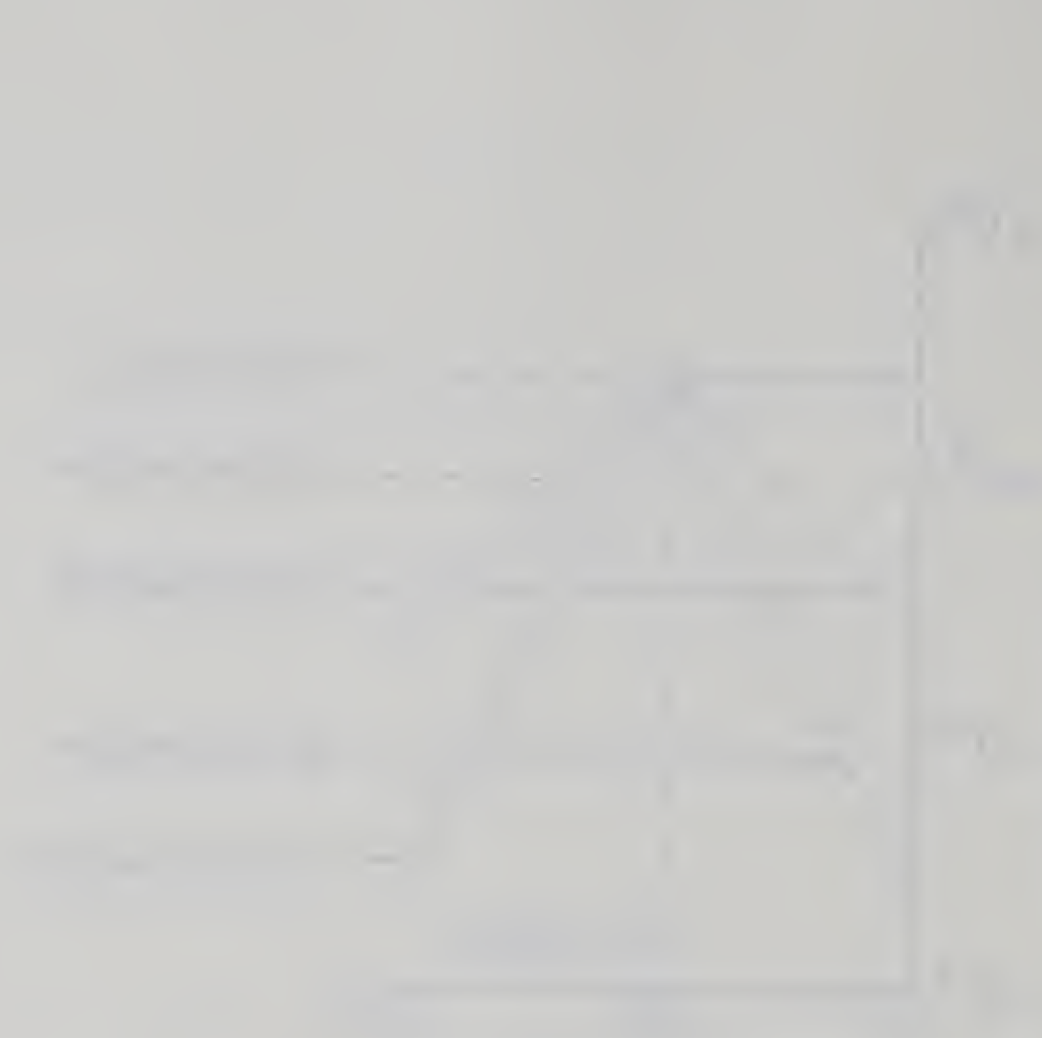


Figure 4-2 The asymptotes of the amplitude response and the phase response of the seismometer to various inputs in ground motion. Units for the amplitude response are volts/(cm/sec^3) for a step in acceleration, volts/(cm/sec^2) for an impulse in acceleration and volts/(cm/sec) for an impulse in ground velocity.



The phase response is shifted forward by 90° with each differentiation of the input signal. Consider the displacement of the suspended mass (ξ) and the displacement of the seismometer mass(x). For low frequencies they are in phase and for high frequencies ξ lags behind x by 180°. If we consider the relative motion of the suspended mass with respect to the seismometer case ($y = x = \xi$, positive y implies downward motion of the suspended mass with respect to the seismometer case), the equation of motion of the suspended mass with no damping (equation 2.1) becomes

$$M\ddot{y} + Ky = M\ddot{x} \ .$$

This can be rewritten as

$$Ky = M\ddot{\xi} \ .$$

The motion of the suspended mass with respect to the seismometer case (y) then is in phase with the acceleration of the suspended mass ($\ddot{\xi}$) which is in phase with the acceleration of the seismometer case (ground acceleration) at low frequencies and lagging behind it by 180° at high frequencies. Since the seismometer is a velocity transducer the output of the seismometer (velocity of the suspended mass with respect to the seismometer case) would then lead the relative displacement by 90°. The output would then lead the ground

acceleration by 90° at low frequencies and follow it by 90° at high frequencies for an impulse in acceleration. For an impulse in velocity the seismometer output then leads the ground velocity by 180° at low frequencies and is in phase with ground velocity at high frequencies. The form of input is determined by the signal put through the calibration coils. A step in current through the coils is equivalent to a step in ground acceleration.

Varying the natural frequency (f_n) of the seismometer changes the output by moving it to the left or right along the frequency scale as the asymptotes always cross at the natural frequency. The load resistance used for a given damping is a function of the natural frequency of the seismometer so a variation in frequency without changing the load resistance would change the damping of the seismometer. The natural frequency of the seismometer is varied by varying the tension on a "sixth spoke" attached to the magnet (Willmore et al, 1963) and thus varying the spring constant, k, of the seismometer. The tension is varied by a screw adjustment on top of the seismometer case.

4.3 Mathematical Treatment of the Data

The input signal, seismometer output and LVDT output are recorded for four cases: no external damping resistor (no electrical damping) and damping resistances of 10 kilohms, 4.6 kilohms and 4.11 kilohms. The recorded data have a dynamic range of ±10 volts in 14 bits plus sign. A further factor of 4 is introduced by the program for changing the data from sequential storage into one block per channel storage. Ten volts is then represented by 32,768.

Any D.C. offset introduced into the seismometer output by the amplifier or data recording subsystem is removed by averaging a number of points at the beginning of the record before the input signal occurs and at the end of the record after the signal has been damped out. The average is then removed from the digitized data. This does not have to be done with the input signal as it is used only to determine the initial condition t_o (the starting point of the response to the input step) or with the LVDT output as this is only used to find the steady state offset ($\Delta\ell$) of the suspended mass. Not removing the offset from the seismometer output is equivalent to adding a step in the time domain over the length of the record and would therefore offset the transform of the data.

The output signal from the unterminated seismometer (no load resistance) is used to find the natural frequency of the seismometer. The expression of the output signal in the time domain due to a step in acceleration is

$$e(t) = \frac{A\Delta\ell\omega_n}{(1-\zeta^2)^{\frac{1}{2}}} e^{-\zeta\omega_n t} \sin \omega_n (1-\zeta^2)^{\frac{1}{2}} t$$

(the inverse Laplace transform of equation 2.7). Measuring the location and height of the peaks yields values for the frequency ($\omega_o = (1-\zeta^2)^{\frac{1}{2}}\omega_n$) and decay rate ($e^{-\zeta\omega_n t}$) of the output signal. It is then possible to calculate the natural frequency ($f_n = 2\pi\omega_n$) and the damping (ζ) of the seismometer.

The output signals from the seismometer with the load resistors are transformed from the time domain to the frequency domain by using z-transforms (subroutines POLYEV, POLAR and DRUM; Robinson, 1967; z-transforms Kanasewich, 1973). The z-transform operates in the time domain and can be used at specific frequencies. We are then able to choose frequencies at equal intervals on a log scale, a desirable attribute as the log amplitude and phase are both plotted vs. log frequency. Frequencies are chosen so there are twenty points per decade between specified low and high frequencies.

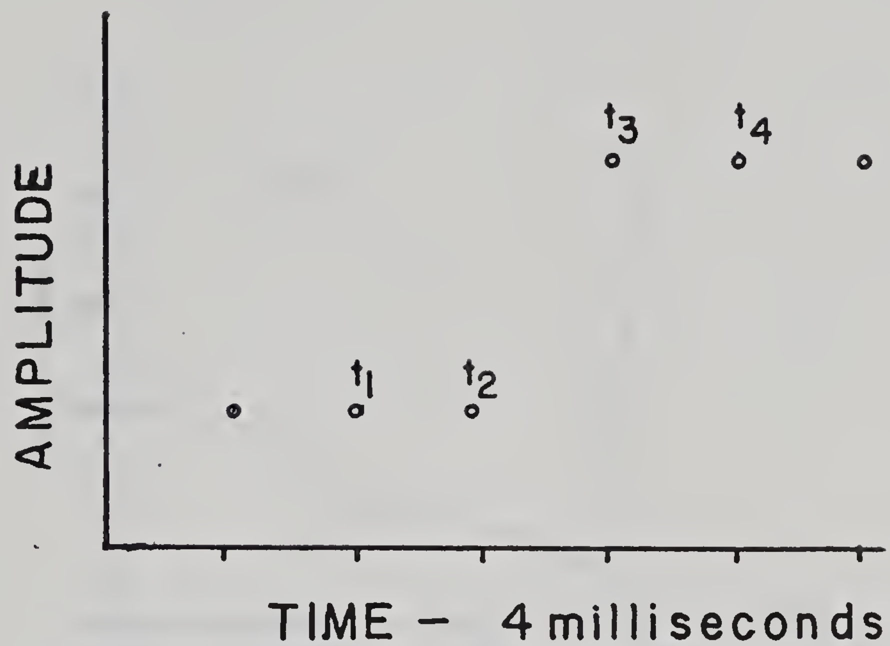


Figure 4-3 The input step digitized at 4 millisecond intervals.

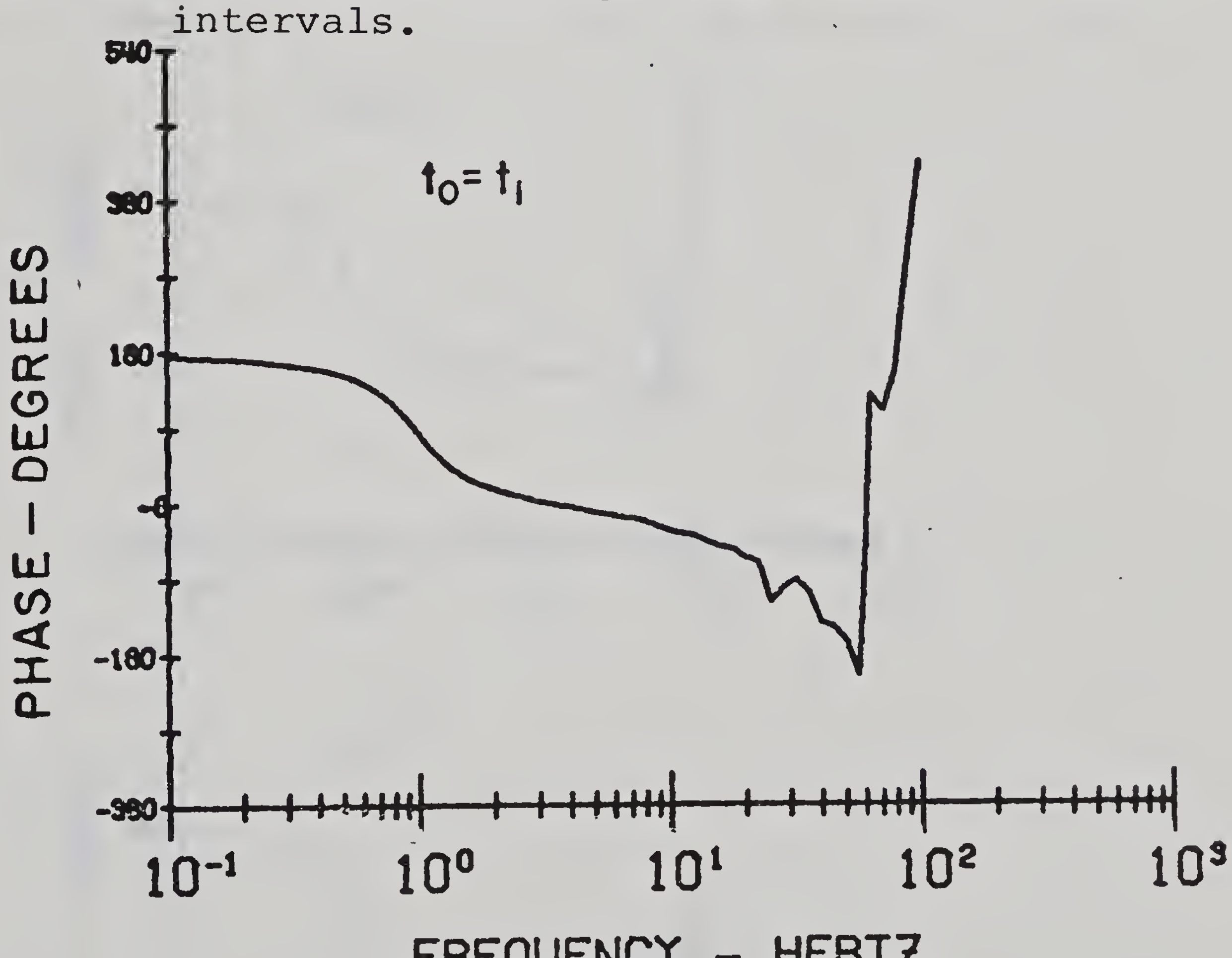


Figure 4-4a The phase response of the seismometer with a 10 kilohm load resistor digitized at intervals of 4 millisecond and having a Nyquist frequency of 125 Hz. The starting point of transformation, t_o, is taken to be t_1.

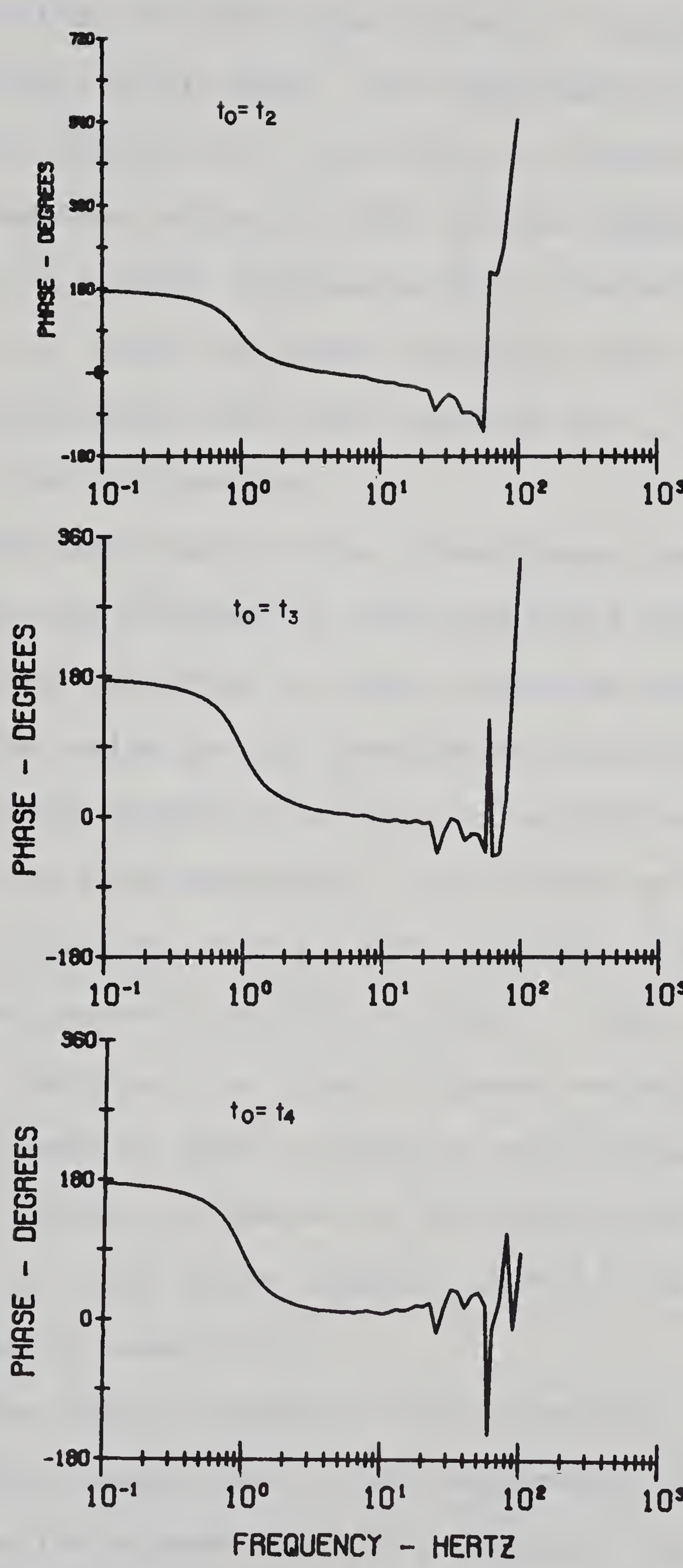


Figure 4-4 b-d The phase response of the seismometer with a Nyquist frequency of 125 hz using values of t_o equal to t_2, t_3 and t_4.

The starting point of the transformation, t_o is determined for the input signal. Figure 4-3 shows the recorded input step. The dependence of phase on the point chosen for t_o is shown in figure 4-4. The phase response varies by 180° at the Nyquist frequency (125 Hz) for every successive point chosen as t_o. Figure 4.4c shows the phase response which uses the first point after the step occurred as t_o; the value used in the calibration.

The amplitude of the transformed data at each frequency was divided by the calculated gain of the seismometer amplifier at that frequency and by the calculated value for $\Delta\ell$ (determined by dividing the steady state change in volts of the LVDT output; by the gain of the LVDT amplifier, 110 ± 1; and by the sensitivity of the LVDT; 7.70 ± 0.003 volts/cm). The amplitude is now expressed in volts/(cm/sec^3). This is the seismometer response to a step in ground acceleration. The amplitude is then rotated by multiplying each point of the frequency response by the normalized frequency response at that point squared ($(f/f_n)^2$) to arrive at the velocity sensitivity.

The phase response is corrected for the calculated phase change due to the seismometer amplifier and delay introduced since all channels are not digitized simultaneously. This introduces a continuous

phase shift which reaches 360° at 2500 Hz (2500 = 1/.0004 where .0004 is the delay between channels) or a shift of 18° at 125 Hz. The phase shift is changed to one due to an impulse in velocity by shifting it forward by 180°. The log amplitude and phase are then plotted against log frequency.

4.4 Observed Results

Calibration runs were done on a Willmore Mk II seismometer, serial number 56450/229518. The runs are done using step and sine wave inputs in current through the calibrating coils, and using the Maxwell bridge. The data from the step input are digitized and processed as in section 4.3. The output from the other two methods is read off an oscilloscope.

a) A Step in Input through the Calibration Coils

A step in current through the calibration coils is used to apply a force to the suspended magnet within the seismometer. This step in magnetic field is equivalent to a step in the inertial frame which is equivalent to a step in ground acceleration. Figure 4-5 shows the log amplitude response observed due to a step in ground acceleration (step acc. sens.) for a 4.1 kilohm load resistor and the velocity sensitivity of the seismometer calculated from it. The digitizing interval used was 0.004 seconds yielding a Nyquist frequency of 125 Hz

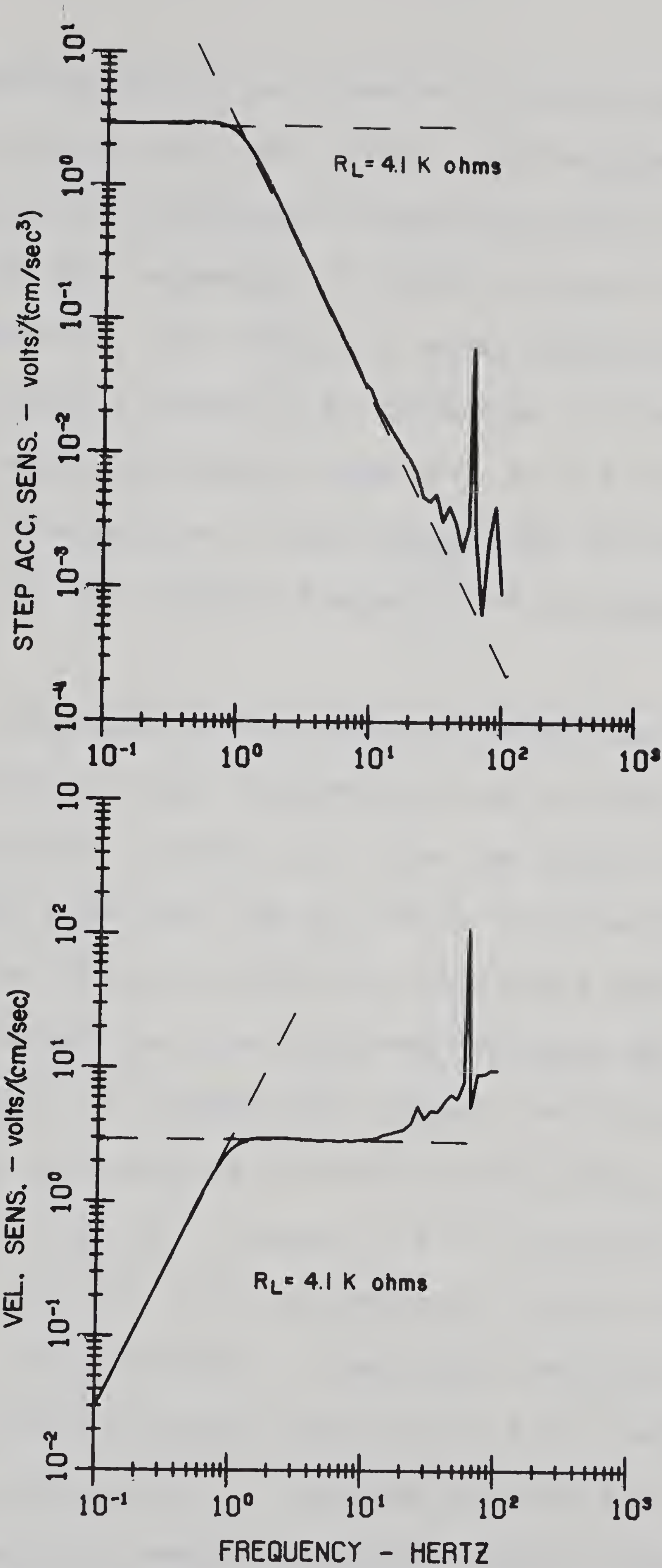


Figure 4-5 The seismometer response to a step in ground acceleration and the calculated velocity sensitivity for a seismometer with a load resistance of 4.1 kilohms. The data is digitized at an interval of 4 milliseconds. The horizontal asymptote has a value of 2.9.

The asymptotes behave as expected in each case - one asymptote of the step acc. sens. plot decreases at 40 db/decade at the higher frequencies while at lower frequencies the asymptote is level at about 2.9 volts/(cm/sec^3). The velocity sensitivity plot has an asymptote with a slope of 40 db/decade at lower frequencies and a horizontal asymptote at 2.9 volt/(cm/sec) at higher frequencies. Both asymptotes in each case intersect at the natural frequency of the seismometer (0.98 Hz).

An increase in the observed values from the expected values at high frequencies can be seen on each of the two graphs in figure 4-5. As the corner of the seismometer amplifier was at 100 Hz and the Nyquist frequency at which the data was digitized was 125 Hz it was believed that the observed increase may be due to alliasing. To lessen this effect the data was digitized at an interval of 0.0008 seconds giving a Nyquist frequency of 625 Hz. Figures 4-6 to 4-8 show the log amplitude curves for the seismometer output signal digitized at this interval. Comparing the step acceleration sensitivity and velocity sensitivity for a seismometer with a damping resistor of 4.1 kilohms (figure 4-8) with the step acceleration sensitivity and velocity sensitivity of the seismometer with the same damping but digitized at 0.004 second interval (figure 4-5),shows no significant change

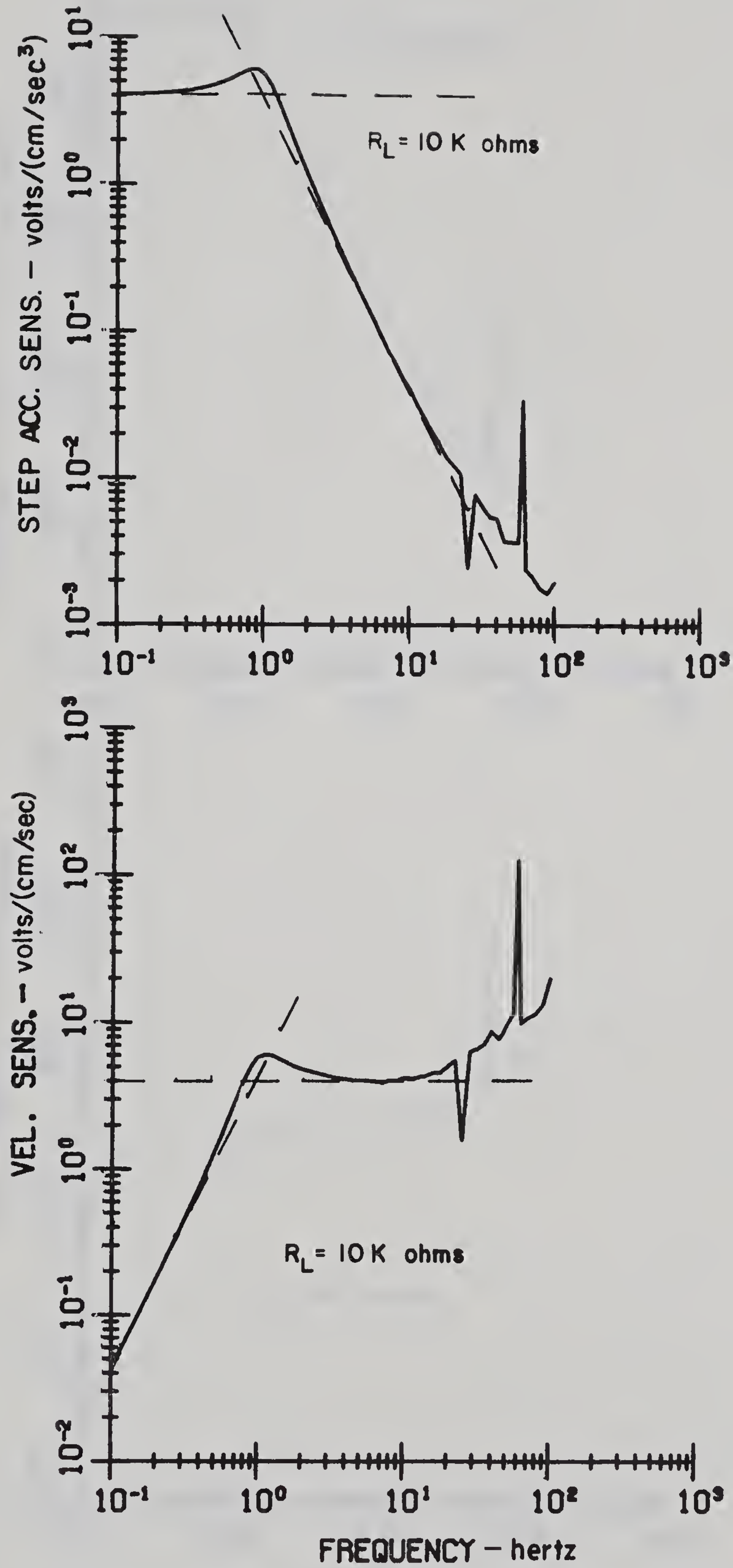


Figure 4-6 The seismometer response to a step in ground acceleration and the calculated velocity sensitivity of a seismometer with a damping resistance of 10 K. The digitizing interval, Δt, is .0008 seconds. The asymptotes are indicated by a dashed line.

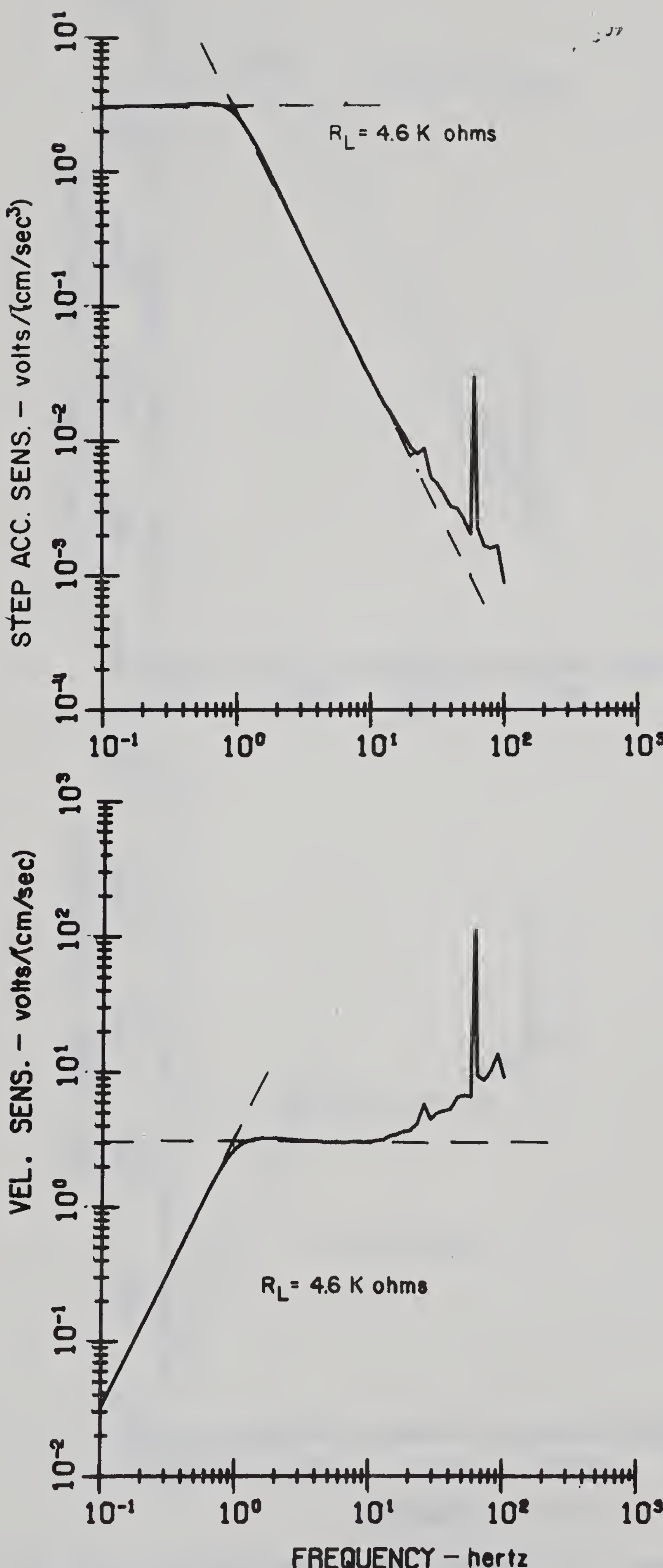


Figure 4-7 The seismometer response to a step in ground acceleration and the calculated velocity sensitivity of the seismometer, Δt = .0008 seconds.

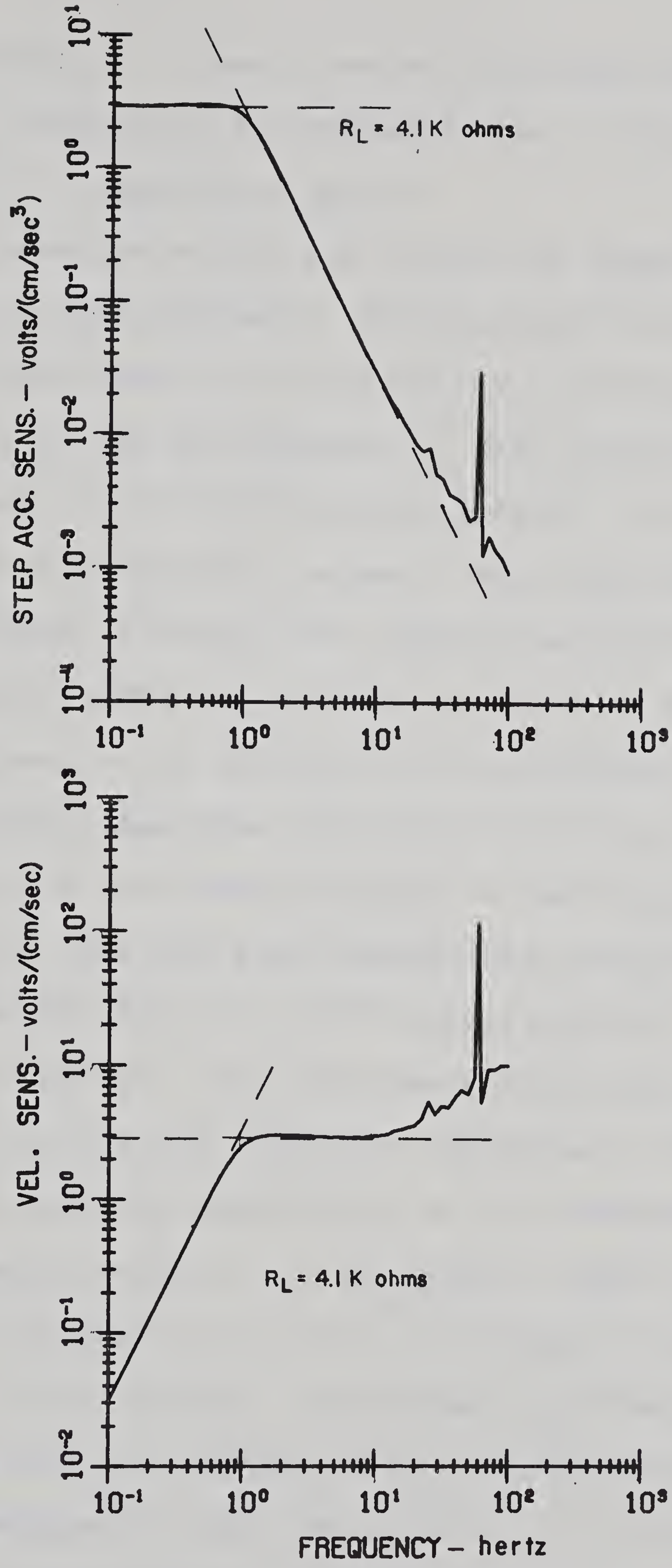


Figure 4-8 The seismometer response to a step in ground acceleration and the calculated velocity sensitivity for a seismometer with a 4.1 K damping resistor Δt = .0008.

in the outputs. It must then be concluded that the observed increase is not primarily due to alliasing but rather to some other source.

A possible source can be seen by examining the size of this increase. The round off of the data in digitizing gives us a possible error of the size of half a bit on the A-D converter. This would result in an increase of 1.98×10^{-3} volts/(cm/sec^3) in the step acceleration sensitivity values. When the values are differentiated to obtain the velocity sensitivity this introduces an error of .8 volts/(cm/sec) at 20 Hz, 3.2 volts/(cm/sec) at 40 Hz and 12.8 volts/(cm/sec) at 80 Hz. Values less than this are all that are needed to account for the noted increase in the velocity sensitivity. The observed step acceleration sensitivity shows an increase of about 1.4×10^{-3} volts/(cm/sec^3) over the expected response. This falls well within the quantization error of the A-D converter (Plešinger, 1971).

The velocity sensitivity of the seismometer with a damping resistor of 10 kilhms shows a horizontal asymptote of 4.05 volts/(cm/sec) and a damping of 0.35 critical (calculated from the graph by reading off the response of the seismometer at the natural frequency). The motor factor, A, of the seismometer is not the value of the horizontal asymptote as the output signal was attenuated by the load

resistance R_c

$$V_{obs} = \frac{R_s}{R_L + R_s} V_{out}$$

where V_{obs} is the observed output, V_{out} is the output from the seismometer, and R_s is the resistance of the coil within the seismometer. The motor factor of the seismometer is then R_s+R_L/R_L times the value of the horizontal asymptote. The measured seismometer coil resistance is 3.68 kilohms. This yields a motor factor of 5.53 volts/(cm/sec) for the 10 kilohm load resistor. The 4.6 kilohm load resistor shows a damping of 0.45 critical and horizontal asymptote of 3.05 volts/(cm/sec) yielding a motor factor of 5.49 volts/(cm/sec). A damping of 0.62 critical and horizontal asymptote of 2.9 volts/(cm/sec) are observed for the 4.1 kilohm load resistor giving a motor factor of 5.52 volts/(cm/sec); this yields an average motor factor of 5.51 ± 0.01 volts/(cm/sec) for the seismometer. The observed damping indicated that 0.707 critical damping requires a load resistance slightly less than 4.1 kilohms and shows the error in the method of calculating the damping resistance.

Figure 4-9 shows the velocity phase response of the seismometer with different damping resistance. The phase behaves as expected, leading the ground velocity by 180° at low frequencies and being in phase with it at high frequencies. The phase response above 1.8 Hz

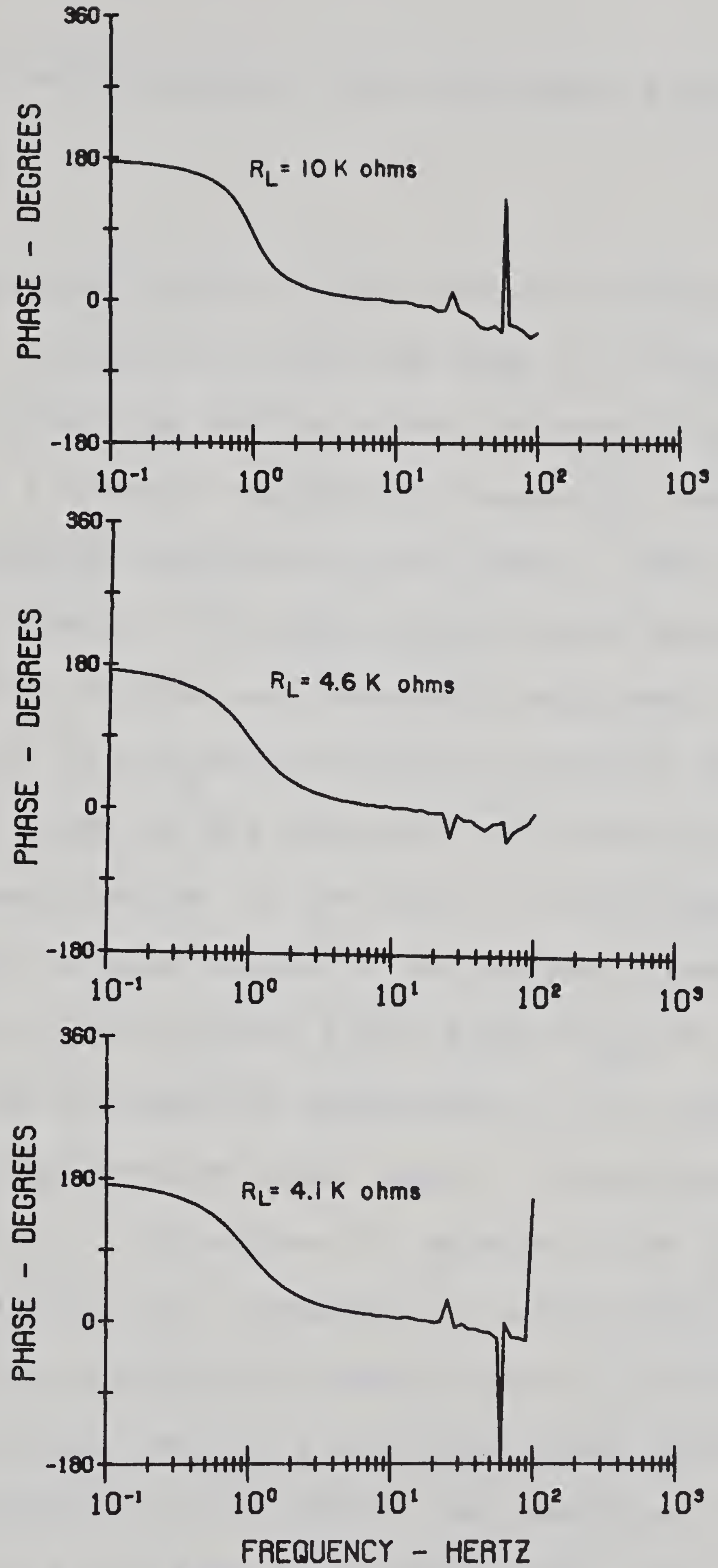


Figure 4-9 Velocity phase response of the seismometer for various load resistances.

is not valid because of the decreased signal to noise ratio.

b) A Sinewave Input through the Calibrating Coils

A calibration run was done by putting a sinewave through the coil at given frequencies and measuring the frequency response by measuring the height of the sinewave observed in the output. The phase was measured using a variable phase signal generator. Comparison of the two frequency responses would show any error in the step calibration method due to the assumed shape of the step, the digitization of the data, the transformation of the data or alliasing. Reading the peak to peak values of the output sinewave and dividing by the steady state deflection of the magnet yield the seismometer sensitivity to an impulse in ground acceleration (acc. sens.). The steady state deflection of the magnet is obtained from a graph of the deflection vs. frequency on a log scale (figure 4-10). The steady state deflection for the seismometer mass is 0.0127 cm for a 10 kilohm load resistance, 0.0148 cm with a 4.6 kilohm load resistance and 0.0145 cm with a 4.1 kilohm load resistance. The acceleration sensitivity of the seismometer calculated using these values is shown in figure 4-11 and the velocity sensitivity

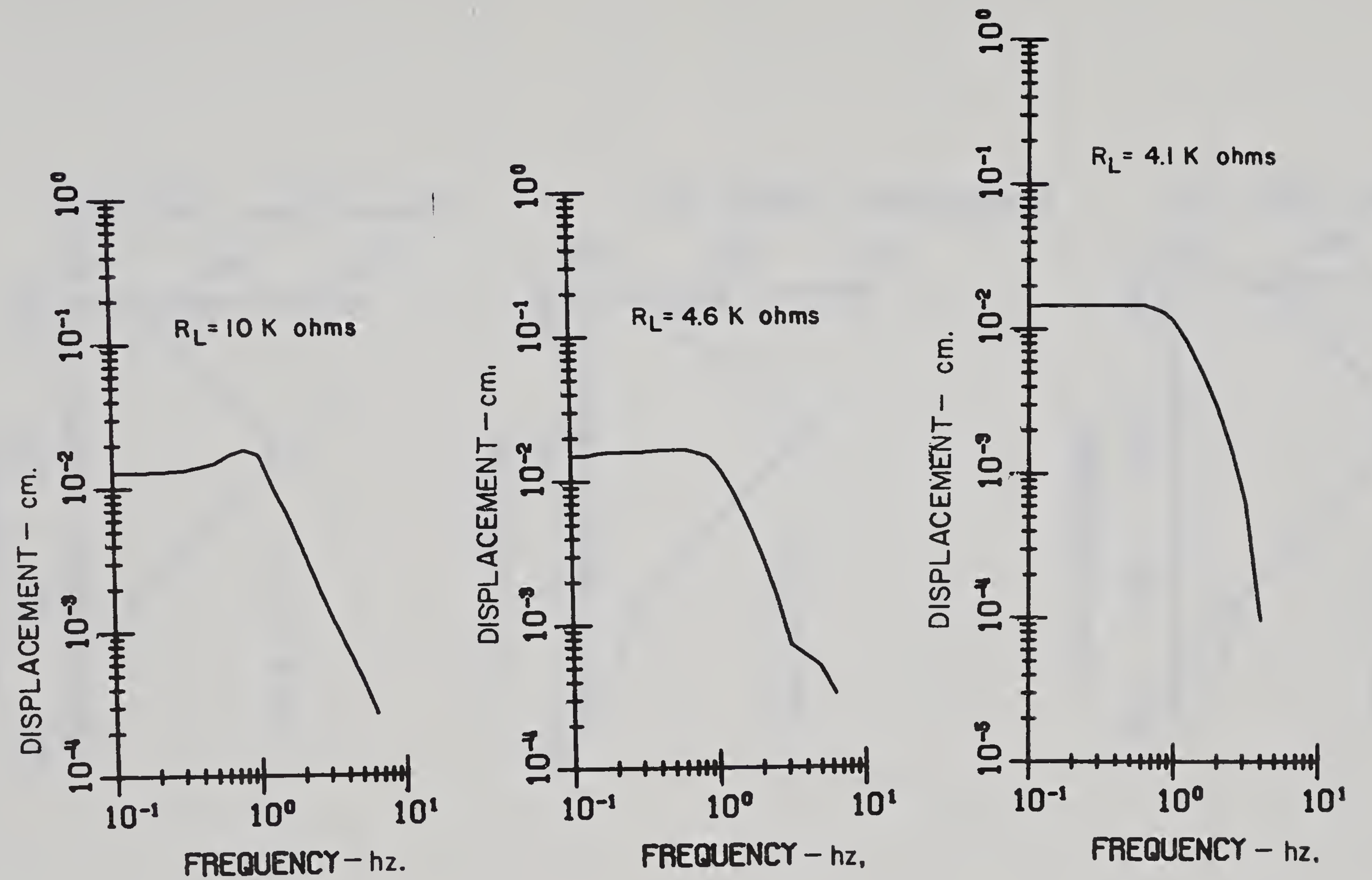


Figure 4-10 The peak to peak displacement of the seismometer mass due to a sine-wave input at several frequencies.

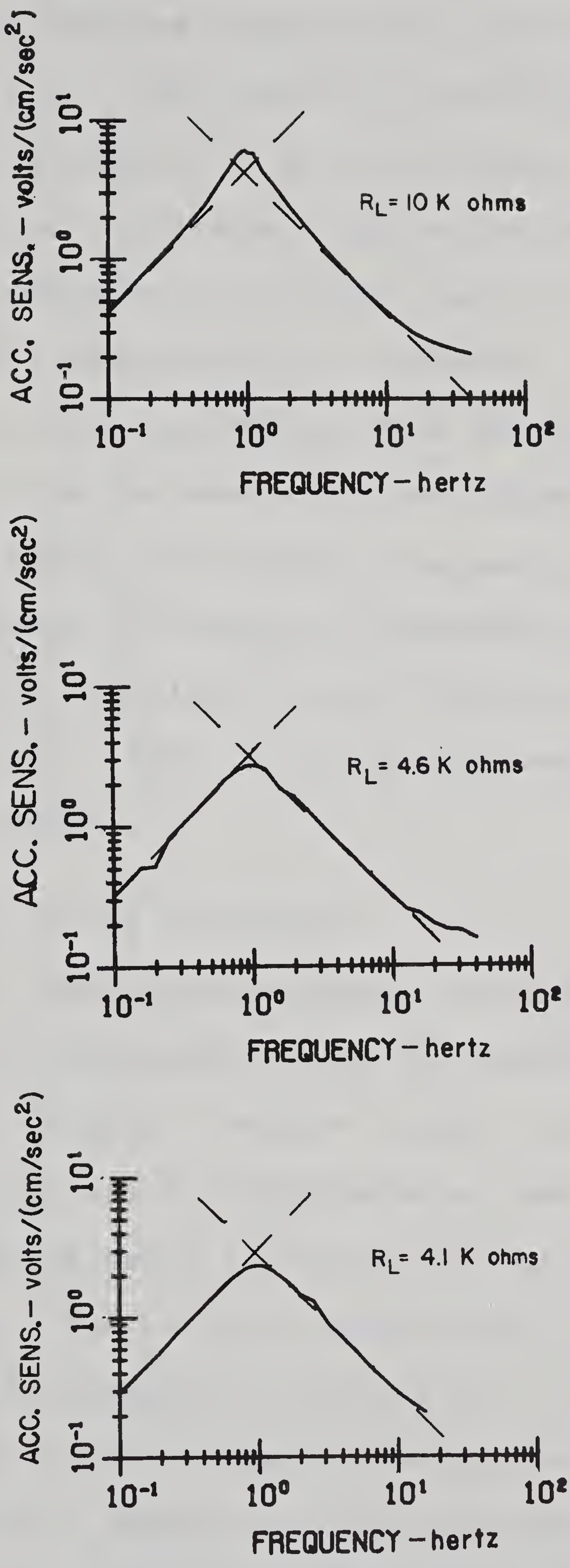


Figure 4-11 Response of the seismometer to an impulse in ground acceleration. Asymptotes are indicated by a dashed line. The natural frequency is .98 hz.

calculated from the acceleration sensitivity is shown in figure 4-12. The velocity sensitivity shows horizontal asymptotes of 4.10 volts/(cm/sec) for a load resistance of 10 kilohms, 3.10 volts/(cm/sec) for a load resistance of 4.6 kilohms, and 2.9 volts/(cm/sec) for a load resistance of 4.1 kilohms. These values agree with those arrived at from the step input. An overlay of the two sets of curves shows an exact agreement within the useable frequency band. The measured phase is shifted to represent the phase between the measured signal and ground velocity by adding 90° (figure 4-13). This too is in agreement with the calculated values.

c) Maxwell Bridge Technique

For comparison purposes a calibration run was made on the seismometer using the Maxwell bridge technique (Chapter 1 section 1.2d). Load resistances of 10 kilohms and 4.1 kilohms were used. The results were evaluated using a program written by Dr. E.R. Kanasewich. The velocity sensitivity arrived at by this method is shown in figure 4-14. The 4.1 kilohm resistor yielded a horizontal asymptote of 2.8 volts/(cm/sec) and a damping of 0.63 critical. The motor factor of the seismometer calculated by this technique was 5.72 volts/(cm/sec). The 10 kilohm resistor had

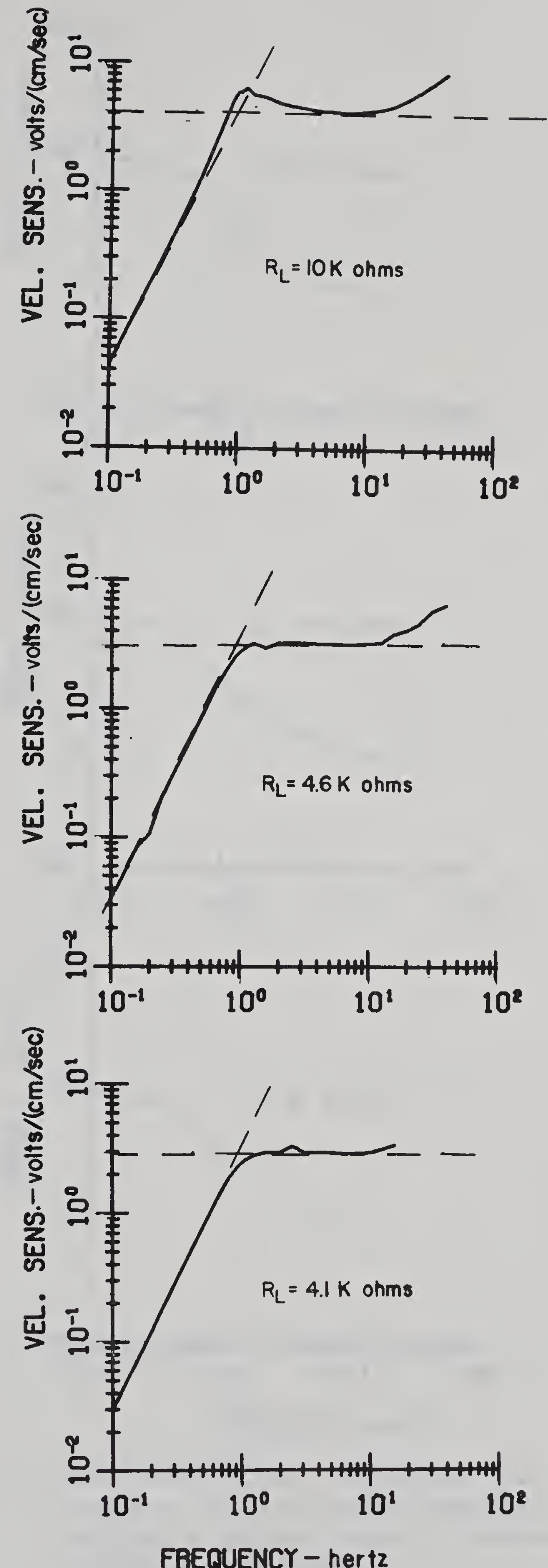


Figure 4-12 The velocity sensitivity of the seismometer with various load resistances for a sine-wave input. The asymptotes are indicated by dashed lines.

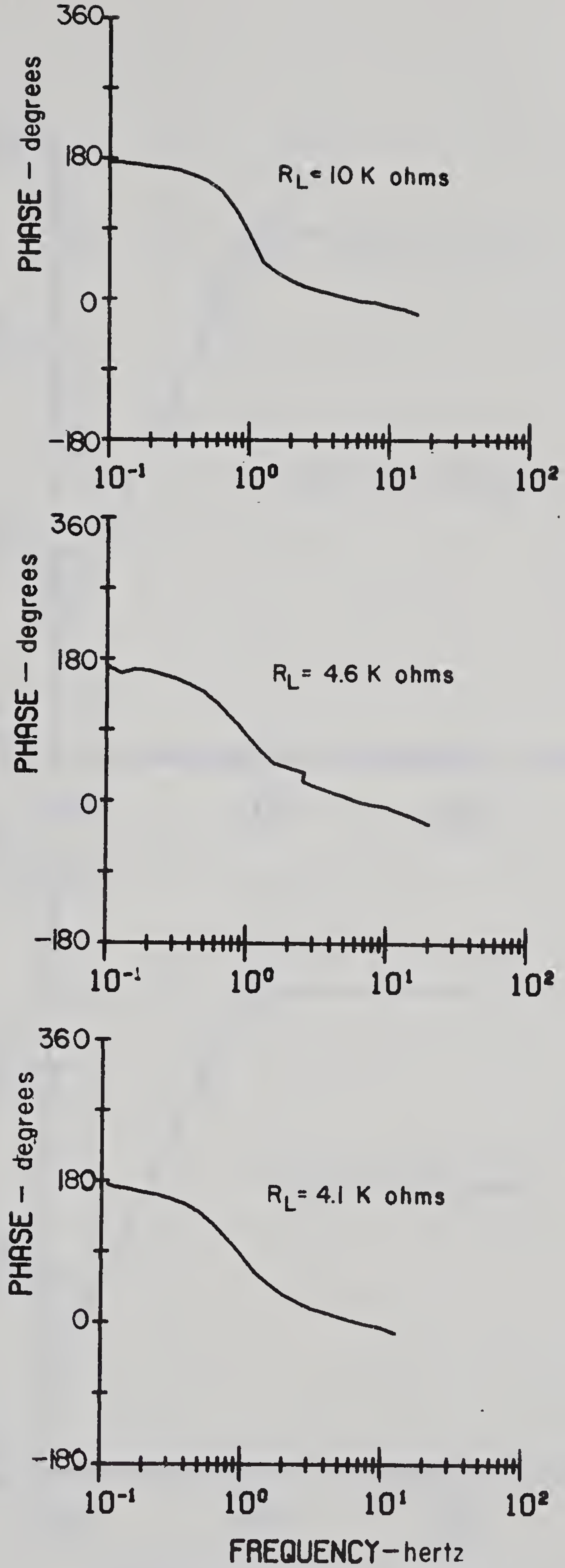


Figure 4-13 Velocity phase response for various damping resistances measured using a variable phase signal generator and a sine-wave input.

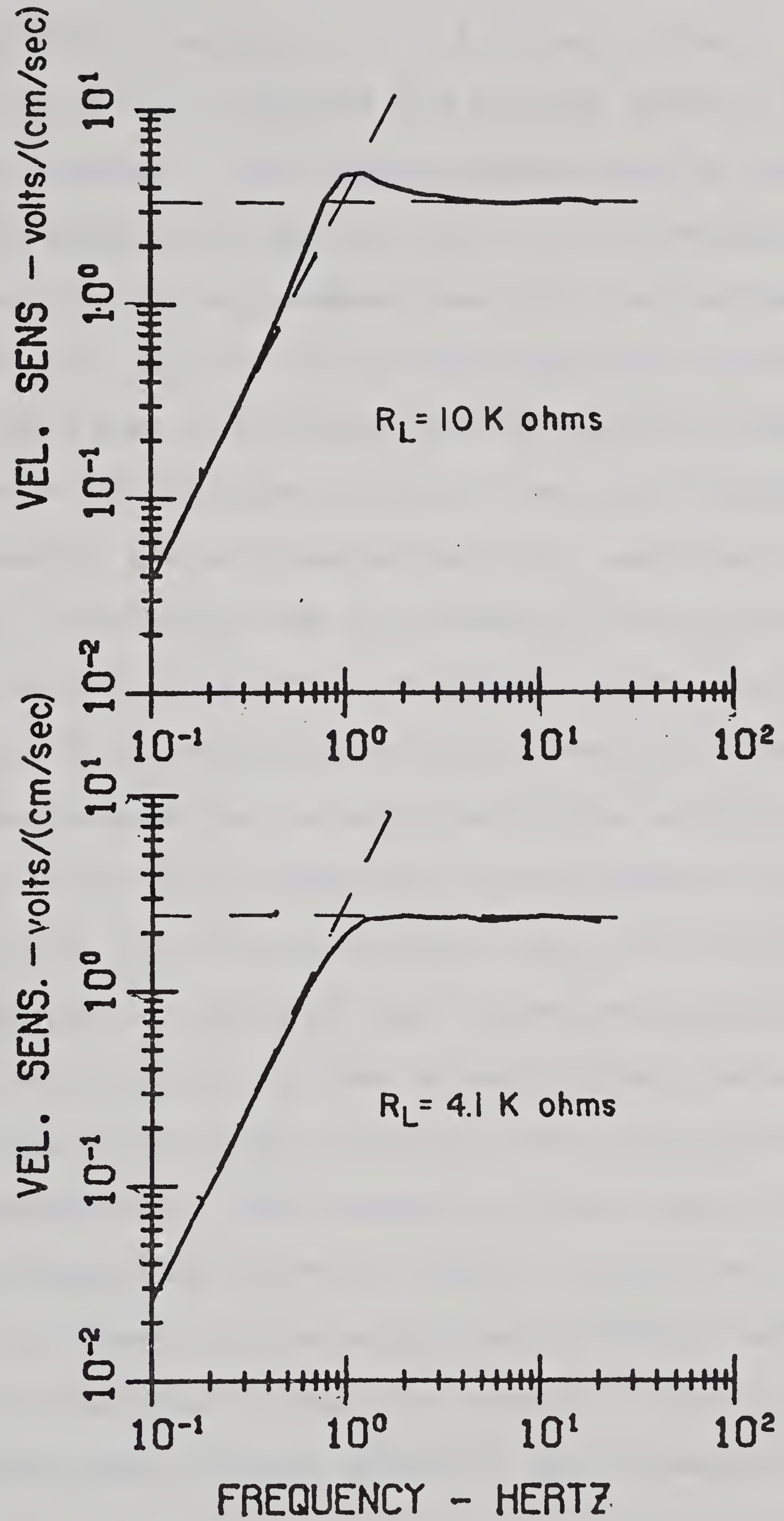


Figure 4-14 The velocity sensitivity of the seismometer arrived at using the Maxwell bridge calibration technique (Willmore, 1959).

a horizontal asymptote of 3.4 volts/(cm/sec), a damping of 0.36 critical and a motor factor of 6.01 volts/(cm/sec). The damping change due to the use of the Maxwell bridge is that the load resistance as seen by the seismometer was not the load resistance put on the amplifier but rather the load resistance of the amplifier in parallel with a variable resistance of about 200 kilohms in size. The load resistance as seen by the seismometer is then less than the values used (1.0 kilohms and 4.1 kilohms) and so the signal is more heavily damped than the cases using the calibration coils. The difference in the values of the motor factor of the seismometer as measured by the calibration coils (5.51 0.01 volts/(cm/sec)) and the Maxwell bridge technique (5.72 volts/(cm/sec) and 6.01 volts/(cm/sec)) may be due to the fact that the Maxwell bridge calibration was carried out over a month after the calibration with the calibrating coils and the motor factor of the seismometer may have changed in this time but the disagreement in the motor factor for the two runs done on the Maxwell bridge using two different damping resistances indicates a possible source of error is the reading taken by this method or by the method itself.

4.5 Conclusion

A step in current through the calibrating coils provides a rapid, accurate method of calibrating any moving magnet seismometer. Though the calibration run was done only on a vertical seismometer neither the direction nor magnitude of the applied force depends on gravity. The calibration technique is then independent of the orientation of the seismometer and calibrating coils with respect to the earth.

Alliasing and digitizing errors involved in the transformation of the data into the frequency domain can be minimized by using a least squares fit on the data to determine the natural frequency, damping and value of the horizontal asymptote of the seismometer response. The velocity sensitivity can be found by substituting these values into the correct expression.

Whether the velocity sensitivity is found using a least squares fit or transforming the data into the frequency domain this method is rapid as it only requires the input of one signal, a square wave and the digitizing of the seismometer output. The sine-wave technique requires readings at a large number of frequencies to ensure accuracy of data so is a time consuming method and therefore not easily applicable to frequent calibration of seismometers. The simplicity

[illegible]

of the step input is ideal in that it can be generated by a clock D-A converter and so no signal generator is needed.

The linearity of the deflection of the seismometer magnet with respect to input current makes it unnecessary to measure the magnet deflection once the coils are mounted on the seismometer case and an initial calibration is done. A remeasurement of deflection is only necessary if a sizable change is noticed in the motor factor of the seismometer. Only one linear variable differential transformer is needed to calibrate all seismometers.

As no contact with the seismometer is needed this technique is easily used for frequent field calibrations where the seismometer may be buried to reduce the effects of pressure and temperature variation. This method then has advantages of the traditional calibration techniques such as Willmore's use of the Maxwell bridge and the use of shake tables.

Allsopp, D.F., M.D. Burke and G.L. Cumming, A Digital Seismic Recording System: Bull. Seism. Soc. Am., 62, No. 6, 1641-1647 (1972).

Bardeen, Thomas, Crystal Shaketable For Geophone Studies: Geophys. 15, No. 3, 456-461 (1950).

Barr, K.G., A Rapid Method For Calibrating Willmore Seismographs: Bull. Seism. Soc. Am. 54, No. 5, Part A, 1473-1477 (1964).

Bogert, Bruce P., Correction Of Seismograms For The Transfer Function Of The Seismometer: Bull. Seism. Soc. Am. 52, No. 4, 781-792 (1962).

Burke, M.D., E.R. Kanasewich, J.D. Malinsky and J.F. Montalbetti, A Wide-Band Digital Seismograph System: Bull. Seism. Soc. Am. 60, No. 5, 1417-1426 (1970).

Dix, G. Hewitt, Seismic Prospecting For Oil: Harpers and Brothers, Chapter 15, pp. 374.

Espinosa, A.F., G.H. Sutton and H.J. Miller, S.J.: Bull. Seism. Soc. Am. 52, No. 4, 767-779 (1962).

Friedler, J., Calibration Of Electromagnetic Seismographs By Means Of Responses To Release And Shock Tests: Studia geoph. et geod. 16 (1972).

Hilger and Watto Ltd., London, England, Instruction Manual For The Willmore Seismometer MKII (1964).

Hoover, D.B., D. Plouff and J.H. Healy, Calibration Of A Seismic-Refraction System: U.S. Geological Survey, Denver, Colorado.

Jarosh, H. and A.R. Curtis, A Note On The Calibration Of The Electromagnetic Seismograph: Bull. Seism. Soc. Am. 63, No. 3, 1145-1155 (1973).

Kanasewich, E.R., Time Sequence Analysis In Geophysics, The University of Alberta Press (1973).

Kanasewich, E.R., W.P. Siewert, M.D. Burke, G.H. McCloughan And L. Ramsdell, Grain-Ranging Analog Or Digital Seismic System: Bull. Seism. Soc. Am. 64, No. 1, 103-113 (1974).

Kollor, F. and R.D. Russell, Seismometer Analysis Using An Electric Current Analog: Bull. Seism. Soc. Am., 56, No. 6, 1193-1205 (1966).

Matumoto, T., Calibration of An Electromagnetic Seismographs By Means Of The Frequency Analysis: Bull. of the Earthquake Res. Inst. 36, 55-64 (1958).

Mitchell, B.J. and M. Landisman, Electromagnetic Seismograph Constants By Least-Squares Inversion: Bull. Seism. Soc. Am. 59, No. 3, 1335-1348 (1969).

Murphy, L.M., Roger M. Wilson, L.R. Burgess and T.H. Pearce, Response Curves Of An Electromagnetic Seismograph By Sine-Wave Simulator Method: Bull. Seism. Soc. Am. 44, 7-20 (1954).

Plešinger, Axel, Two Identification Methods And Their Application To Seismograph System Calibration: Institute of Radio Engineering and Electrons, Czechoslovak Academy of Sciences, Praha, Půre No. 47 (1971).

Robinson, Enders A., Multichannel Time Series Analysis With Digital Computer Programs: Holden-Day, San Francisco (1967).

Russell, R.D., R.D. Meldrum and O.G. Jenson, The Maxwell Bridge As A Circuit Element In Electromagnetic Feedback Seismographs: Bull. Seism. Soc. Am. 58 No. 5, 1621=1630 (1968).

Shima, Etsuzo and Robert P. Mayer, Total System Calibration of High-Frequency Seismograph Systems By Use Of The Maxwell Bridge; In The Earth Beneath The Continents, Am. Geophys. Union Geophys. Monograph 10, 538-59 (1966).

Shima, Etsuzo, Theoretical And Experimental Approach To The Designs and Calibrations Of Electro-Magnetic Seismograph: I Voltage Sensitivity Of The Moving-Coil Type Seismomoeter: Bull. of the Earthquake Res. Inst. 38, 29-39 (1960).

Stewart, I. and D.J. Sutton, An Electromagnetic Calibration Coil For A Willmore MKII Seismometer: J. Sci. Instrum. 44 (1967).

Sutton, D.J., A Note On Phase Shift In Electromagnetic Seismographs: Earthquake Notes 39, 23-28 (1968).

White, R.E., Modifications To Willmore's Method For Calibrating Seismographs Using A Maxwell Bridge: Bull. Seism. Soc. Am. 60, No. 6, 2015-2022 (1970).

Wickens, A.J., H.S. Hasegawa, and M.N. Bone, Pulse Calibration And Its Application To The Daily Calibration Of The Canadian Standard Seismograph Network Long Period Seismometers: Can. J. Earth Sci. 11, 691-697 (1973).

Willmore, P.L., The Application Of The Maxwell Impedance Bridge To The Calibration Of Electromagnetic Seismographs: Bull. Seism. Soc. Am. 49, 99-112 (1959).

Willmore, P.A. and D.V. Connell, A New Short-Period Seismometer For Field And Observatory Use: Bull. Seism. Soc. Am. 53, No. 4, 835-844 (1963).